MAESTRO

CALCULUS 1

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Differentiation

$$y = f(x) \Longrightarrow y' = f'(x) = \frac{dy}{dx}$$

* General Rules For differentiation :

	$f(x)$	$f'(x)$
1	x^n	$n x^{n-1}$
	x^3	$3x^2$
	x^{-3}	$-3x^{-4}$
	$\frac{1}{x^2} = x^{-2}$	$-2x^{-3} = \frac{-2}{x^3}$
	$\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$	$-\frac{2}{3} x^{-\frac{5}{3}} = \frac{-2x^{-\frac{5}{3}}}{3} = \frac{-2}{3x^{\frac{5}{3}}}$
2	C (constant)	0
	5	0
3	$C \cdot F(x)$	$C \cdot f'(x)$
	$5x^3$	$5 \cdot 3x^2 = 15x^2$
	$10x^3$	$30x^2$
4	$f \pm g$	$f' \pm g'$
	$3x^2 + 4x - 5$	$6x + 4$
5	$a^{F(x)}$	$\underset{\text{نفس}}{a^{F(x)}} \cdot \underset{\text{مشتقة الاس}}{f'(x)} \cdot \ln a$
	3^{x^2}	$3x^{2} \cdot 2x \cdot \ln 3$
	e^{3-x^2}	$e^{3-x^2} \cdot -2x \cdot \overset{1}{\cancel{\ln e}} = -2xe^{3-x^2}$
	e^x	$e^x \cdot 1 \cdot 1 = e^x$

6	فقط للجذر التربيعي وليس الزوجي ويجب أن يكون في البسط $\sqrt{F(x)}$	$\frac{f'(x)}{2\sqrt{F(x)}}$ ← مشتقة ما داخل الجذر
	$\sqrt{x^2+3x}$	$\frac{2x+3}{2\sqrt{x^2+3x}}$
	$y = \sqrt{3^x+x}$	$y' = \frac{3^x \cdot \ln(3) + 1}{2\sqrt{3^x+x}}$
	important $\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
7	$\ln F(x)$	$\frac{f'(x)}{F(x)}$ ← مشتقة ما داخل الـ ln
	$\ln x^3$	$\frac{3x^2}{x^3} = \frac{3}{x}$
	$\ln(3x^2+4)$	$\frac{6x}{3x^2+4}$
	important $\ln x$	$\frac{1}{x}$
$\ln x = \log_e x$ 8	$\log_a F(x)$	$\frac{f'(x)}{F(x) \cdot \ln a}$ ← الأساس
	$\log_2(3x^2+4)$	$\frac{6x}{(3x^2+4)\ln 2}$
9	cos	- sin . مشتقة الزاوية
	sin	cos . مشتقة الزاوية
	tan	$\sec^2$. مشتقة الزاوية
	cot	- csc . مشتقة الزاوية
	sec	tan . sec . مشتقة الزاوية
	csc	- csc . cot . مشتقة الزاوية
	- csc	csc . cot . مشتقة الزاوية

Q_1 $F(x) = \sin x^2$, Find $f'(x)$

$$f'(x) = \cos x^2 . 2x$$

Q_2 $F(x) = \tan\sqrt{x} + \sec(\ln x)$, Find $F'(x)$

$$f'(x) = \sec^2\sqrt{x} \cdot \underset{\text{مشتقة الزاوية}}{\frac{1}{2\sqrt{x}}} + \sec(\ln x) . \tan(\ln x) \cdot \underset{\text{مشتقة الزاوية}}{\frac{1}{x}}$$

Q_3 $F(x) = \ln(3^x + x^2) + \sqrt{5x+2} + \frac{1}{2\sqrt{x}}$, Find $F'(x)$

$$f'(x) = \frac{3^x . 1 . \ln 3 + 2x}{3^x + x^2} + \frac{5}{2\sqrt{5x+2}} + \frac{1}{2}\left(-\frac{1}{2} x^{-\frac{3}{2}}\right)$$

	$f(x)$	$f'(x)$
10	$\sin^{-} x$	$\frac{1}{\sqrt{1-(x)^2}}$. مشتقة الزاوية
	$\cos^{-} x$	$\frac{-1}{\sqrt{1-(x)^2}}$. مشتقة الزاوية
	$\tan^{-} x$	$\frac{1}{1+(x)^2}$. مشتقة الزاوية
	$\cot^{-} x$	$\frac{-1}{1+(x)^2}$. مشتقة الزاوية
	$\sec^{-} x$	$\frac{1}{x\sqrt{(x)^2-1}}$. مشتقة الزاوية
	$\csc^{-} x$	$\frac{-1}{x\sqrt{(x)^2-1}}$. مشتقة الزاوية
11	$f . g$	$f g' + g f'$
	$\sqrt{x} . \ln x$	$\sqrt{x} \cdot \frac{1}{x} + \ln x \cdot \frac{1}{2\sqrt{x}}$
12	$\frac{f}{g}$	$\frac{g f' - f g'}{g^2}$
	$\frac{3^x}{x^2+1}$	$\frac{\overset{\text{مقام}}{(x^2+1)} . \overset{\text{مشتقة البسط}}{3^x . \ln 3} - \overset{\text{بسط}}{3^x} \overset{\text{مشتقة المقام}}{(2x)}}{\underset{\text{مقام}}{(x^2+1)^2}}$

Q₁ $f(x)=\sin^{-} x^2$, Find $f'(x)$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \underline{\underline{2x}} \rightarrow$$ مشتقة الزاوية

Q₂ $f(x) = \sec^{-}(\ln x)$

$$f'(x) = \frac{1}{\ln x\sqrt{(\ln x)^2-1}} \cdot \underline{\underline{\frac{1}{x}}} \rightarrow$$ مشتقة الزاوية

Q₃ $f(x) = \tan^{-} \sqrt{x}$

$$f'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \left\| \frac{1}{2\sqrt{x}} \right\| \rightarrow$$ مشتقة الزاوية

Q₄ $f(x) = x^2+3x-2$, Find $\boxed{f'(1)} \rightarrow$ معناها اشتق ثم عوض بالواحد

$$f'(x) = 2x+3$$

$$f'(1) = 2(1)+3 = 5$$

Q₅ $y = 3^x + \sqrt{\cos 3x}$, find $\boxed{\left.\frac{dy}{dx}\right|_{x=0}} \rightarrow$ معناها اشتق ثم عوض بالصفر

$$y' = 3^x \cdot 1 \cdot \ln 3 + \frac{-\sin 3x \cdot 3}{2\sqrt{\cos 3x}}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \ln 3 + \frac{0}{2} = \ln 3$$

Q₆ $y = x^2-2x+1$, $x=3$, $dx = 0.2$

Find dy

$$\frac{dy}{dx} = 2x-2 \Longrightarrow \frac{dy}{0.2} = 6-2$$

$$\frac{dy}{0.2} = 4 \Longrightarrow dy = 0.8$$

Q7 Find $\frac{d}{dx}\overset{y}{(\tan x^2+2^x)}$

$y = \tan x^2 + 2^x$

$\frac{dy}{dx} = \sec^2 x^2 + 2^x (2x) 1 . \ln 2 = \sec^2 x^2 . 2x + 2^x \ln 2$

Q8 $y = 5x^2 + 3x - 1$, Find Value of x that makes the derivative of $F(x) = \underset{y'}{-17}$

$y' = 10x + 3 = -17$

$-17 = 10x + 3 \Longrightarrow \frac{-20}{10} = \frac{\cancel{10}x}{\cancel{10}}$

$\boxed{\therefore\ x = -10}$

Q9 $F(x) = \ln \sqrt[3]{\frac{x+1}{x-1}}$, find $f'(x)$ Note $\rightarrow$ $\ln a - \ln b = \ln \frac{a}{b}$

$f(x) = \left(\frac{x+1}{x-1}\right)^{\frac{1}{3}} \Longrightarrow F(x) = \frac{1}{3}\ln\left(\frac{x+1}{x-1}\right)$

$= \frac{1}{3}\left(\ln(x+1) - \ln(x-1)\right)$

$\therefore f'(x) = \frac{1}{3}\left(\frac{1}{x+1} - \frac{1}{x-1}\right)$

Q10 $f(x) = x^3 . g(x)$, $g(2) = 3$, $g'(2) = 1$, Find $f'(2)$

$f'(x) = x^3 . g'(x) + g(x) . 3x^2$

$f'(2) = 8 . g'(2) + g(2) . 12$

$= 8 + 3 . 12 = 44$

* high derivativ

$$y = f(x)$$

$$\Downarrow$$

لازم القوس ← $f^{(2)} = f''(x) = \frac{d^2y}{dx^2}$

$$f^{100} = \frac{d^{100}y}{dx^{100}}$$

Q₁ $f(x) = \sec x$. Find $f''(\frac{\pi}{4})$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x . \sec^2 x + \sec x . \tan x . \tan x$

$= \sec(\frac{\pi}{4}) . \sec^2(\frac{\pi}{4}) + \sec(\frac{\pi}{4}) . \tan(\frac{\pi}{4}) . \tan(\frac{\pi}{4})$

$= \sqrt{2} . (\sqrt{2})^2 + \sqrt{2} . 1 . 1 =$

$= 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$

تفاحتين + تفاحة = 3تفاحات

$\sec \frac{\pi}{4} = \sqrt{2}$

$\tan \frac{\pi}{4} = 1$

Q₂ $f(x) = \underbrace{\cancel{3x^{78}} + \cancel{2x^{52}}} + 4x^{40} - \underbrace{\cancel{3x^{18}} + \cancel{2x^{2}} + \cancel{5}}$ → (بعد 40 مرة اشتقاق رح يصيروا صفر)

Find $\left.\frac{d^{40}f}{dx^{40}}\right|_{x=0}$

(لأنه لم نضربهم بصفر رح يروحوا)

$= 4(40x^{39})$

$= 4(40.39\,x^{38})$

$= 4(40.39.38\,x^{37})$

$= 4(40!)$ ← الناتج النهائي

* **Case 1** $\Rightarrow$ Q_1 $F(x) = \sin x$

find $\dfrac{d^{99}\sin x}{dx^{99}} \xRightarrow{\text{هي نفسها}} \boxed{\dfrac{d^3 \sin x}{dx^3}} = -\cos x$

Note

$F(x) = \sin x$

$F'(x) = \cos x$

$F''(x) = -\sin x$

$F'''(x) = \underline{\underline{-\cos x}}$ → هي الاجابه

بنقسم دائماً على 4 لما يكون في sin x, cos x ← 4

$$\begin{array}{r} 24 \\ 4\overline{)\,99} \\ 96 \\ \hline \textcircled{3} \end{array}$$

③ → هادا هو رقم المشتقة الجديد

Q_2 find $\dfrac{d^{41}\cos x}{dx^{41}} = -\sin x$

$$\begin{array}{r} 10 \\ 4\overline{)\,41} \\ 40 \\ \hline \textcircled{1} \end{array}$$

① → هادا رقم المشتقة الجديد لل cos x

Q_3 find $\dfrac{d^{53}\sin 5x}{dx^{53}} = 5^{53}\cdot \cos 5x$

$$\begin{array}{r} 4\overline{)\,53} \\ 52 \\ \hline \textcircled{1} \end{array}$$

① → هادا رقم المشتقة الجديد لل sin5x

Q_4 find $\dfrac{d^{41}\cos 3x}{dx^{41}} = 3^{41}\cdot(-\sin 3x)$

* **Case 2** $\Rightarrow$ Q_1 $F(x) = 2^x$. Find $\dfrac{d^{40}2^x}{dx^{40}} = 2^x\cdot(\ln 2)^{40}$

Q_2 $f(x) = \underline{\underline{\text{arc}}}\tan(3^x)$. Find $f'(x)$

→ $\tan^{-}(3^x)$

$F'(x) = \dfrac{1}{1+(3^x)^2}\cdot 3^x \ln 3$

مشتقة الزاوية

Chain Rules

First Rule →

	$f(x)$	$f'(x)$
1 -	$Fog(x) = F(g(x))$	$F'(g(x)) \cdot g'(x)$
2 -	$\sin(x^2)$	$\cos(x^2) \cdot 2X$
3 -	$\tan(3x+2)$	$\sec^2(3x+2) \cdot 3$

Q_1 $F'(2) = 3$, $g'(1) = 4$, $g(1) = 2$

Find $\overset{\text{طريقة (1)}}{\frac{d\,Fog(x)}{dx}}$ أو $\overset{\text{طريقة (2)}}{g'(Fog(x))}$ أو $\overset{\text{طريقة (3)}}{y = Fog(x)}$, Find $\frac{dy}{dx}\Big|_{x=1}$

$$Fog(x) = f(g(x))$$

$$\frac{d\,f(g(x))}{dx} = f'(g(x)) \cdot g'(x)$$

$$\frac{dF(x)}{dx} = f'(g(1)) \cdot g'(1)$$

$$= f'(2) \cdot 4 = 3 \cdot 4 = 12$$

Q_2 $\frac{d\,Fog(3)}{dx} = 5$, $f'(3) = 4$, $g(3) = 3$, $F(3) = 3$

Find $g'(3)$

$$f'(g(3)) \cdot g'(3) = 5$$

$$g'(3) \cdot 4 = 5 \implies \frac{\cancel{4}g'(3)}{\cancel{4}} = \frac{5}{4} \implies \therefore g'(3) = \frac{5}{4}$$

$\underline{\underline{Q_3}}$ $y = f(x^2+1)$, $f'(2) = 2$, $f(2) = 3$

Find $\left.\frac{dy}{dx}\right|_{x=1}$

$y' = f'(x^2+1) \cdot 2x$

$y' = f'(2) \cdot 2(1) \implies y' = 2 \cdot 2 = 4$

$\underline{\underline{Q_4}}$ $f(2x+1) = 3x^2 + 2x$, Find $f'(5)$

$f'(\underline{\underline{2x+1}}) \cdot 2 = 6x + 2$

→ لازم نخليها = 5

$f'(2(2)+1) \cdot 2 = 6(2) + 2$

$2f'(5) = 12 + 2$

$\frac{\cancel{2}f'(5)}{\cancel{2}} = \frac{14}{2} \implies f'(5) = 7$

مهم **$\underline{\underline{Q_5}}$** $\frac{df(2x+1)}{dx} = 3x^2 + 2x$, Find $f'(5)$

$f'(2x+1) \cdot 2 = 3x^2 + 2x$

$f'(2(2)+1) \cdot 2 = 3(2)^2 + 2(2)$

$2f'(5) = 12 + 4$

$\frac{\cancel{2}f'(5)}{\cancel{2}} = \frac{16}{2}$

$f'(5) = 8$

Q_6 $g(x) = F(6x + h(x))$, $h(1) = 2$, $F'(8) = 4$, $h'(1) = 0$

find $g'(1)$

$$g'(x) = f'(6x + h(x)) \cdot 6 + h'(x)$$

$$g'(1) = f'(6(1) + h(1)) \cdot 6 + h'(1)$$

$$= f'(6 + 2) \cdot 6 + 0$$

$$= f'(8) \cdot 6 \Longrightarrow 4 \cdot 6 = 24$$

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second Rule $\longrightarrow$

	$F(x)$	$F'(x)$
1-	$(F(x))^n$	$n(F(x))^{n-1} \cdot F'(x)$
2-	$(3x+2)^2$	$2(3x+2) \cdot 3$
3-	$(x^3+2^x)^{-2}$	$-2(x^3+2^x)^{-3} \cdot 3x^2 + 2^x \cdot \ln 2$ $\Downarrow$ $= \dfrac{-2(3x^2+2^x \cdot \ln 2)}{(x^3+2^x)^3}$

Q_1 $y = \sin^3(x^2)$, Find $\dfrac{dy}{dx}$

$$y = (\sin(x^2))^3$$

$$\frac{dy}{dx} = 3(\sin x^2)^2 \cdot \cos x^2 \cdot 2x$$

$$\frac{dy}{dx} = 3\sin^2 x^2 \cdot \cos x^2 \cdot 2x$$

$\underline{\underline{Q_2}}$ $g(x) = \sin^2 x + \cos x^2$, find $g'(x)$

$$g(x) = (\sin x)^2 + \cos x^2$$

$$g'(x) = 2(\sin x) \cdot \cos x \cdot 1 + (-\sin x^2) \cdot 2x$$

$\underline{\underline{Q_3}}$ $y = (\arcsin 3x)^2$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2(\arcsin \underline{\underline{3x}}) \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot \underline{\underline{3}}$$

مشتقة الزاوية

$\underline{\underline{Q_4}}$ $f^3(x) = 8x^2$, find $f'(1)$ given $f(1) = 2$

$$(f(x))^3 = 8x^2$$

$$3(f(x))^2 \cdot f'(x) = 16x$$

$$3(f(1))^2 \cdot f'(1) = 16(1)$$

$$3(2)^2 \cdot f'(1) = 16$$

$$\frac{\cancel{12}f'(1)}{\cancel{12}} = \frac{16}{12}$$

$$f'(1) = \frac{16}{12}$$

Third Rule

Third Rule $\longrightarrow$ $\boxed{\begin{matrix} m = 3U+2 \\ \frac{dm}{dU} = 3 \end{matrix}}$

$$\boxed{\frac{dy}{dx} = \frac{dy}{dU} \cdot \frac{dU}{dx}}$$

$$\boxed{\frac{dm}{dU} = \frac{dm}{dX} \cdot \frac{dX}{dU}}$$

Q₁ $y = \sin U \quad , \quad x = U^2$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \underset{\downarrow}{\frac{dy}{dU}} \cdot \underset{\downarrow}{\frac{dU}{dx}}$$

$\cos U$ $\qquad$ $\frac{dx}{dU} = \frac{2U}{1}$

$\downarrow$

$\frac{dU}{dx} = \frac{1}{2U}$

$\therefore \frac{dy}{dx} = \frac{\cos U}{2U}$

Q₂ $m = 3x^2 + 2 \quad , \quad t = 3x+1$

Find $\frac{dt}{dm}$

$$\frac{dt}{dm} = \underset{\downarrow}{\frac{dt}{dX}} \cdot \underset{\downarrow}{\frac{dX}{dm}}$$

3 $\qquad$ $\frac{dm}{dx} = 6X$

$\downarrow$

$\frac{dX}{dm} = \frac{1}{6X}$

$\therefore \frac{dt}{dm} = \frac{3}{6X} = \frac{1}{3X}$

Q₃ $\left.\frac{dy}{dU}\right|_{v=2} = 1 \quad , \quad U = x^3 + 1$

Find $\left.\frac{dy}{dx}\right|_{x=1}$

$$\frac{dy}{dx} = \underset{\downarrow}{\frac{dy}{dU}} \cdot \underset{\downarrow}{\frac{dU}{dX}}$$

$1 \qquad 3x^2$

$\therefore \frac{dy}{dx} = 1 \cdot 3x^2$

$= 1 \cdot 3(1)^2 = 3$

Q₄ $y = \sqrt{x^2+1} \quad , \quad U = \sin^{-}x$

Find $\left.\frac{dy}{dv}\right|_{x=0}$

$$\frac{dy}{dv} = \underset{\downarrow}{\frac{dy}{dX}} \cdot \underset{\downarrow}{\frac{dX}{dv}}$$

$\frac{2x+1}{2\sqrt{x^2+1}}$ $\qquad$ $\frac{dv}{dx} = \frac{1}{\sqrt{1-(x)^2}}$

$\downarrow$

$\frac{dx}{dv} = \sqrt{1-(x)^2}$

عوض بالصفر

$\therefore \frac{dy}{dx} = \frac{2x+1}{2\sqrt{x^2+1}} \cdot \frac{\sqrt{1-(x)^2}}{1} = \frac{0}{\sqrt{1}} \cdot \sqrt{1} = 0$

Q_5 $y = (x^2+3x)^2$, $U = 2x+3$, find $\frac{dy}{d}\Big|_{U=1}$

ملحوظة هذي هي قيمة U وليس قيمة X لذلك نجد قيمة X بدلالة الـ U

$U = 2x+3$

$\because U = 1$

$\therefore 1 = 2x+3$

$1 - 3 = 2x$

$\frac{-\cancel{2}}{\cancel{2}} = \frac{\cancel{2}x}{\cancel{2}}$

$\boxed{\therefore x = -1}$

$$\frac{dy}{dU} = \frac{dy}{dx} \cdot \frac{dx}{dU}$$

$$= 2(x^2+3x).(2x+3)$$

$$\frac{du}{dx} = 2 \quad \therefore \frac{dx}{dU} = \frac{1}{2}$$

$$\frac{dy}{dU} = \cancel{2}(x^2+3x).(2x+3).\frac{1}{\cancel{2}}$$

$$\therefore \frac{dy}{dU}\Big|_{x=-1} = (-1)^2+3(-1).2(-1)+3$$

$$= 1 - 3 = -2$$

Implicit Differentiation

$$y \longrightarrow y' \longrightarrow \frac{dy}{dx}$$

- $h(x) = (F(x))^3 \Longrightarrow h'(x) = 3(F(x))^2 \cdot F'(x)$
- $h(x) = y^3 \Longrightarrow h'(x) = 3y^2 \cdot \underline{\underline{y'}}$ → مهم كتير

Q₁ $x^2+y^2+3x = xy$ Find $\frac{dy}{dx}$ at $(\overset{x}{1}, \overset{y}{1})$

$$2x + 2y \cdot y' + 3 = xy' + y(1)$$

$$2(1) + 2(1) \cdot y' + 3 = (1)y' + (1)$$

$$2 + 2y' + 3 = y' + 1$$

$$2y' - y' = 1 - 2 - 3$$

$$y' = -4$$

Q₂ $x^2 + 3x + 2 = \sin(xy)$, find y'

$$\frac{2x+3}{\cos(xy)} = \frac{\cancel{\cos(xy)} \cdot xy' + y}{\cancel{\cos(xy)}}$$

$$\frac{2x+3}{\cos(xy)} = xy' + y \Longrightarrow \therefore \frac{\cancel{x}y'}{\cancel{x}} = \frac{\frac{2x+3}{\cos(xy)} - y}{x}$$

Q_3 $f(x) + x^2 . (f(x))^3 = 10$, $f(1) = 2$ find $f'(1)$

$f'(x) + x^2 . 3(f(x))^2 . f'(x) + (f(x))^3 . 2x = 0$

$f'(1) + (1)^2 . 3(f(1))^2 . f'(1) + (f(1))^3 . 2(1) = 0$

$f'(1) + 12\, f'(1) + 16 = 0$

$\frac{\cancel{13}\, f'(1)}{\cancel{13}} = -\frac{16}{13}$

$f'(1) = \frac{-16}{13}$

Q_4 $g(x) + x.\sin(g(x)) = x^2$, $g(0) = \frac{\pi}{2}$, find $g'(0)$

$g'(x) + \underline{\underline{\overset{=0}{\overset{\uparrow}{x}}}}\, c\cancel{\overset{0}{os\,(g(x))}} . g'(x) + \sin(g(x)) = 2x$

$g'(0) + \sin(g(0)) = 0$

$g'(0) = -\sin(\frac{\pi}{2}) \Longrightarrow \quad g'(0) = -1$

Q_5 $x^2y + xy + y + y^2 = 10x$, find y' at $(\overset{x}{0}, \overset{y}{1})$

$\cancel{\overset{0}{x^2}y'} + \cancel{y(2\overset{0}{x})} + \cancel{\overset{0}{x}y'} + y + y' + 2y.y' = 10$

$(1) + y' + 2(1).y' = 10$

$1 + y' + 2y' = 10$

$\frac{\cancel{3}y'}{\cancel{3}} = \frac{9}{3} \Longrightarrow \therefore y' = 3$

Q_6 $\ln y = x^3 + 3x$, find $\frac{dy}{dx}$

$\cancel{yx}\frac{y'}{\cancel{y}} = 3x^2 + 3 \quad xy$

$y' = y(3x^2 + 3)$

Q_7 $y = (x^2+1)^x$, Find y'

ملحوظة: إذا طلب مشتقة (اقتران)^اقتران نستخدم طريقة (ln)

نضع ln للطرفين

$\ln y = \ln(x^2+1)^x$

$\ln y = x\ln(x^2+1)$

$yx\,\frac{y'}{y} = \left(x \cdot \frac{2x}{x^2+1} + \ln(x^2+1)\cdot(1)\right) \underset{\downarrow}{x}y$

اشارة ضرب وليس مجهول

$y' = y\left(\frac{2x^2}{x^2+1} + \ln(x^2+1)\right)$

$(x^2+1)^x \longrightarrow$ شوية زناخة 😉

Q_8 $y = (\sin x)^x$, find y'

$\ln y = \ln(\sin x)^x$

$\ln y = x\ln(\sin x)$

$\cancel{xy}\,\frac{y'}{\cancel{y}} = \left(x \cdot \frac{\cos x}{\sin x} + \overset{\ln}{\uparrow}\sin x\right)\ xy$

$y' = \underset{\searrow(\sin x)^x}{y}\left(x\cot + \ln\sin x\right)$

Q_9 $y = (\sec x^2)^{\sqrt{x}}$, find y'

$\ln y = \ln(\sec x^2)^{\sqrt{x}}$

$\ln y = \sqrt{x}\cdot\ln(\sec x^2)$

$\cancel{y}x\,\frac{y'}{\cancel{y}} = \left(\sqrt{x}\cdot\frac{\cancel{\sec x^2}\cdot\tan x^2}{\cancel{\sec x^2}}\,\underline{\underline{2x}} + \ln(\sec x^2)\cdot\frac{1}{2\sqrt{x}}\right)^{xy}$

$y' = y\left(\sqrt{x}\cdot 2x\tan x^2 + \frac{\ln(\sec x^2)}{2\sqrt{x}}\right)$

Q_{10} $y = \frac{x^2 \cdot \sin x \cdot \sqrt{x}}{(x^2+1)^2}$, Find y'

$$\ln y = \ln\left(\frac{x^2 \cdot \sin x \cdot \sqrt{x}}{(x^2+1)^2}\right)$$

$\ln\sqrt{x} = \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x$

$$\ln y = \ln x^2 + \ln \sin x + \ln \boxed{\sqrt{x}} - \ln (x^2+1)^2$$

$y \times$ ← اشارة ضرب

$$\frac{y'}{y} = \frac{2x}{x^2} + \frac{\cos x}{\sin x} + \frac{1}{2} \cdot \frac{1}{x} - (2)\frac{2x}{x^2+1}$$

$\times y$ → اشارة ضرب

$$y' = y\left(\frac{2x}{x^2} + \frac{\cos x}{\sin x} + \frac{1}{2} \cdot \frac{1}{x} - \frac{4x}{x^2+1}\right)$$

Q_{11} $F(x) = g(x) + \frac{x}{g(x)}$, $g'(2) = 2$, $g(2) = 4$

Find $F'(2)$

$$f'(x) = g'(x) + \frac{g(x) - xg'(x)}{(g(x))^2}$$

$$f'(x) = 2 + \cancel{\frac{4 - 2(2)}{16}}$$

$$f'(x) = 2$$

Rule $\Longrightarrow (f^{-}(a))' = \frac{df^{-1}(a)}{d(x)} = \frac{1}{f'(f^{-}(a))}$

Q_1 $f(\underline{3}) = 4$, $f'(3) = \frac{3}{2}$, find $\frac{df^{-}(4)}{dx}$

$$\frac{df^{-}(4)}{dx} = \frac{1}{f'(f^{-}(4))} = \frac{1}{f'(\underline{3})} = \frac{2}{3}$$

مقلوب

Q_2 $f'(2) = 4$, $f(2) = -3$. Find $(f^{-}(-3))'$

$$(f^{-}(-3))' = \frac{1}{f'(f^{-}(-3))} = \frac{1}{f'(2)} = \frac{1}{4}$$

مقلوب

مهم

Q_3 $f(x) = x^2 - 6x - 22$, $x > 1$, Find $\boxed{\frac{d f^{-}(-15)}{dx}}$

$-15 = x^2 - 6x - 22$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$\downarrow$ $\downarrow$

$\underline{\underline{x = 7}}$ $x = -1$

$f'(x) = 2x - 6$

$= 14 - 6 = 8$

$$= \frac{1}{f'(f^{-}(-15))}$$

$$= \frac{1}{f'(7)}$$

$$= \frac{1}{8}$$

الاقتران المتشعب

$$f(x) = \begin{cases} x^2 + 3x + 1 & , x < 1 \\ mx + b & , x \geq 1 \end{cases}$$

قابلة للاشتقاق

f(x) is differentiable at $x = 1$. Find m, b

شرطين

② $\lim_{+} = \lim_{-}$

(متصل)

① $f'_{+} = f'_{-}$

$\lim_{x \to 1^+} mx + b = m + b$

$\lim_{x \to 1} x^2 + 3x + 1 = 5$

$\therefore m + b = 5$

$5 + b = 5$

$\therefore b = 0$

$$f'(x) = \begin{cases} 2x + 3 & , x < 1 \\ m & , x > 1 \end{cases}$$

$\uparrow$ اشارة المساواة تُهمل في الاشتقاق.

$f'^{+}(1) = m$

$f'^{-}(1) = 5$ $\therefore \boxed{m = 5}$

(نقاط عدم الاشتقاق) Not diff pont(s)

$\frac{F}{g}$ أصفار المقام

$|F(x)|$

$F(x) = \{$

رسمة

Q1 $F(x) = \dfrac{x}{\frac{2}{x} - 1}$, F(x) is not diff... at 0,2 ??

$\frac{2}{x} - 1 = 0$

$\frac{2}{x} = 1 \Longrightarrow \therefore \boxed{x = 2} \longrightarrow$ صفر المقام الأول

$\Longrightarrow \boxed{x = 0} \longrightarrow$ صفر المقام الثاني

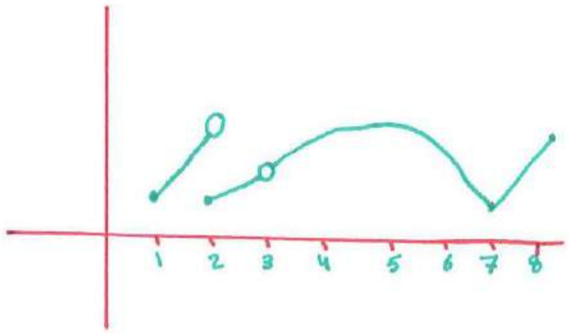
Q2 $F(x) = |3x+6|$ F(x) is not diff at ..... ??

$3x + 6 = 0$

$\frac{3x}{3} = \frac{-6}{3} \Longrightarrow \therefore \boxed{x = -2} \longrightarrow$ الرأس المدبب هو نقطة عدم اتصال

hole Jumps ends

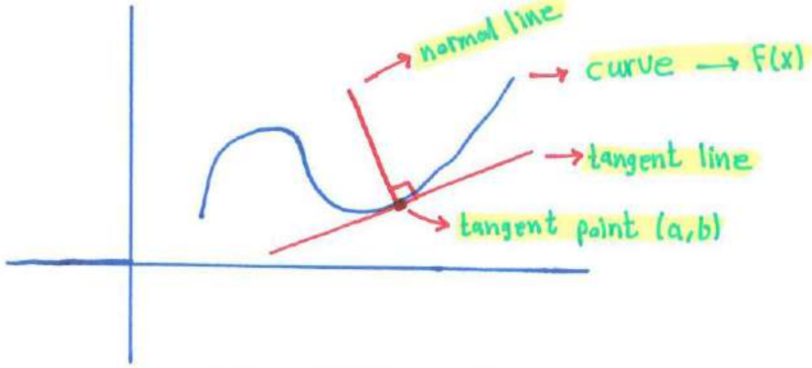
ملحوظة : الاقتران غير قابل للاشتقاق عند الاطراف والانقطاعات و الحلقات والرأس المدبب .



f(x) is not diff at

x = 1 , 3 , 2 , 7
8

Tangent line equation ::



$$\boxed{y - b = m(x-a)}$$

(a,b) → tangent point

$m \longrightarrow$ slope $\longrightarrow f'(a)$

$\hookrightarrow y - b = f'(a)(x-a)$

$\underline{\underline{Q_1}}$ $e^x \cos x$, find tangent line equation at $x=0$

$a = 0 \quad , \quad b = 1 \quad , \quad (0,\underline{\underline{1}})$

$f(0) = e^0 \cos 0 = 1$

$f'(x) = e^x(-\sin x) + \cos x \, e^x$

$f'(0) = 0 + 1 = 1$

$\therefore y - 1 = 1(x-0) \implies y - 1 = \underline{\underline{-1}}(x-0)$

نفس المعادلة الاصلية بس بنقلب وبنغير الاشارة.

$y - 1 = x \qquad\qquad y - 1 = -x$

$y = x + 1 \rightarrow$ tangent line $\qquad y = 1 - x \rightarrow$ Normal line

Note :

$\rightarrow$ two parallel lines // $\rightarrow$ $m_1 = m_2$

$\rightarrow$ two perpendicular lines $\rightarrow$ $m_1 = \frac{3}{2} \rightarrow m_2 = \frac{-2}{3}$

Q_2 $x^2 + 2xy + x = 4$, Find tangent line and normal line eqaution at $\underline{x = 1}$

$\because x = 1 \implies \therefore (1)^2 + 2(1)y + 1 = 4$

$1 + 2y + 1 = 4$

$2 + 2y = 4 \implies \frac{2y}{2} = \frac{2}{2} \implies \therefore \boxed{y = 1}$

$(1, 1)$ $\rightarrow$ tangent points

بنشتق $2x + 2xy' + y.2 + 1 = 0$

$2(1) + 2(1)y' + 2 + 1 = 0$

$2 + 2y' + 3 = 0 \implies 5 + 2y' = 0$

$\frac{2y'}{2} = \frac{-5}{2} \implies \boxed{y' = -\frac{5}{2}}$

$\therefore y - 1 = f'(a)(x - a)$

$y - 1 = \frac{-5}{2}(x - 1) \implies$ tangent line

$y - 1 = \frac{2}{5}(x - 1) \implies$ normal line

$\underline{\underline{Q_3}}$ find value(s) of x that make the slope of

$F(x) = x^2 + 3x + 2$ is horizontal $\xrightarrow{\text{يعني}}$ $F'(x) = 0$

$F'(x) = 2x + 3 = 0$

$2x + 3 = 0 \implies \frac{\cancel{2}x}{\cancel{2}} = -\frac{3}{2} \implies \therefore \boxed{x = \frac{-3}{2}}$

Note :-

١- إذا طلب ← h. tangent نشتق الاقتران ونساوي بسط المشتقة بالصفر.

٢- إذا طلب ← V. tangent نشتق الاقتران ونساوي مقام المشتقة بالصفر.

$\underline{\underline{Q_4}}$ h. tangent , V. tangent $\xrightarrow{\text{For}}$ $F'(x) = \frac{2x}{x^2-4}$

$2x = 0$	$x^2 - 4 = 0$
$\therefore x = 0$	$x = 2 , x = -2$

Note :-

١- اي لدين بالعالم بدي الـ slope تبعه بشتقه.

$\underline{\underline{Q_5}}$ For eqaution of tangent line to the curve $F(x)$ that's perpiendicular to the line $2x + y = 1$ at $(\overset{x}{1}, \overset{y}{2})$

بنعوض فيهم

$2 + y' = 0$

$y' = -2 = m_{ine}$

$\therefore m_t = \frac{1}{2}$

$y - 2 = \frac{1}{2}(x - 1)$

* اذا كان المطلوب في السؤال ايجاد نقطة نتبع الخطوات التالية :

1- نجد ميل المماس $\overset{m_t}{}$ من خلال مشتقة الاقتران (curve)

2- نجد ميل المماس من خلال معادلة خط مستقيم معطاة في السؤال بحيث لو كانت علاقة الخط المستقيم بالمماس parallel يكون $m_L = m_t$ واذا كانت العلاقة تعامد يكون $m_t = \frac{-1}{m_L}$

3- نساوي ميل المماس الناتج من خطوة 1 مع ميل المماس من خطوة 2 ونجد النقطة

Q_6 the curve $y = \tan^{-1} x$ has tangent line parallel to the line $y = 1 + \frac{1}{2}x$ when $x =$

solution $y' = \frac{1}{1+x^2} = m_t$ (from curve)

(from line) $y' = \frac{1}{2} = m_L \parallel m_t$ so $m_t = \frac{1}{2}$

$\rightarrow m_t = m_t$

$\frac{1}{1+x^2} = \frac{1}{2} \rightarrow x^2+1 = 2$

$x = 1, -1$

Q_7 The line $y = x+b$ is tangent to the curve $y = e^x$, find b

(from line) $y' = 1 = m_t$

(from curve) $y' = e^x = m_t$

$e^x = 1$

$\boxed{x = 0}$

$\rightarrow$ now $y = e^x$

$\boxed{y = e^0 = 1}$

$\rightarrow y = x+b$

$1 = 0+b$

$\boxed{b = 1}$

Linear approximation

Note

$$y - f(a) = f'(a)(x-a)$$

$$= \boxed{f(x) = f(a) + f'(a)(x-a)}$$

$\underline{\underline{Q_1}}$ find value of → a) $\sqrt{17}$ b) $\sin 29^\circ$

a) $\sqrt{17}$

$f(x) = \sqrt{x}$

$x = 17$

$a = 16$ → هو عبارة عن رقم بنعرف ناتج تعويضه

$f'(x) = \frac{1}{2\sqrt{x}}$

$\because f(x) = f(a) + f'(a)(x-a)$

تعويض

$\therefore \sqrt{17} = \sqrt{16} + \frac{1}{2\sqrt{16}}(17-16)$

$= 4 + \frac{1}{2 \times 4}$

$= 4 + \frac{1}{8}$

b) $\sin 29^\circ$

$f(x) = \sin x$

$x = 29^\circ$

$a = 30^\circ$

$f'(x) = \cos x$

$\because f(x) = f(a) + f'(a)(x-a)$

$\therefore \sin 29 = \sin 30^\circ + \cos 30^\circ (29^\circ - 30^\circ)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot (-1^\circ) \cdot \underline{\underline{\frac{\pi}{180}}}$

هذه الخطوة للتحويل إلى الدرجات

$= \frac{1}{2} - \frac{\pi\sqrt{3}}{360}$

Q_2 find , $\cos 62^\circ$

$F(X) = \cos X$

$x = 62$

$a = 60$

$F'(x) = -\sin X$

$\because F(X) = F(a) + F'(a)(X-a)$

$\therefore \cos 62 = \cos 60 + (-\sin 60).(2^\circ)$

$= \frac{1}{2} - \frac{\sqrt{3}}{\cancel{2}} . \cancel{2} . \frac{\pi}{180}$

$= \frac{1}{2} - \frac{\pi\sqrt{3}}{180}$

Q3 find linear approximation $f(x) = \sqrt{x}$ about $x = 1$

هي مش قيمة X الحقيقية ، هاي قيمة a .

$f(x) = \sqrt{x}$

$x = ??$

$a = 1$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\Nearrow$

$f(x) = f(a) + f'(a)(x-a)$

$\sqrt{x} = \sqrt{1} + \frac{1}{2\sqrt{1}}(x-1)$

$= 1 + \frac{1}{2}(x-1)$

Q4 find linear appro... for $f(x) = \sqrt{x+3}$ about $x = 13$

هاي مش قيمة X الحقيقية .

$f(x) = \sqrt{x+3}$

$x = ??$

$a = 13$

$f'(x) = \frac{1}{2\sqrt{x+3}}$

$\Nearrow$

$f(x) = f(a) + f'(a)(x-a)$

$\sqrt{x+3} = \sqrt{16} + \frac{1}{2\sqrt{16}}(x-13)$

$= 4 + \frac{1}{8}(x-13)$

ملاحظة :

١- ال a هو عبارة عن عدد أنا بعرف ناتج تعويضه .

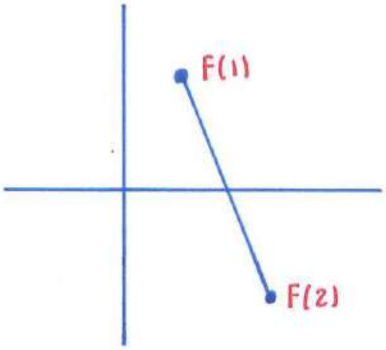
٢- ال X هو عبارة عن عدد أنا مابعرف ناتج تعويضه .

Intermediate value Theorm (IVT)

Q_1 $F(1) = 3$, $F(2) = -2$. How many root ? → solution

أو أو

$F(1) > 0$ $F(2) < 0$



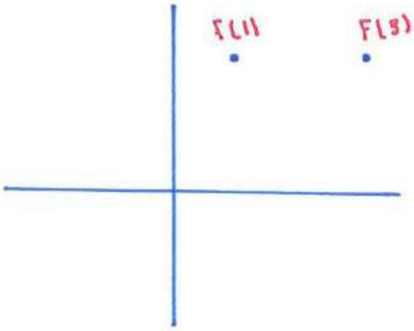
a) at least one root

b) exactly one root

c) at most one root → مابتيجي

ملاحظة : في اي سؤال How many root الجابه "دائماً" at least ، الا إذا ذكر في الاقتران
أنه متزايد أو متناقص .

Q_2 $F(1) > 0$, $F(3) > 0$. $F(x)$ has at least 2 roots , then $F(2)$ could be ?



a) 2

b) 0

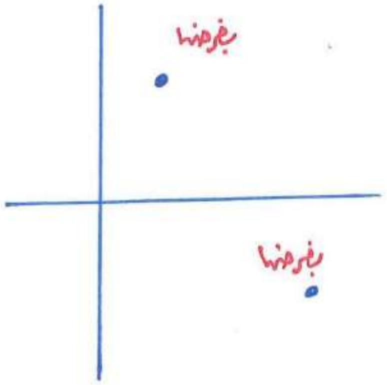
c) 3

d) -4

بنحل السؤال عن طريق التعويض بالخيارات المعطاة .

Q₃ $x^2+5x=-1$, $F(x)$ has at least one root on the interval

$F(x) = x^2+5x+1$



a) (1,3) $F(1)=7$, $F(3)=43$

b) (-2,1) $F(-2)=-5$, $F(1)=7$

c) (,)

d) (.)

في السؤال السابق لازم يكون ناتج التعويض بالرقمين المعطاة واحد سالب وواحد موجب علشان يقدر يقطع محور X-axis ← one root

* Hyperbolic Function :

1) $\sinh x = \frac{e^x - e^{-x}}{2} \longrightarrow$ odd

2) $\cosh x = \frac{e^x + e^{-x}}{2} \longrightarrow$ even

3) $\tanh x = \frac{\sinh x}{\cosh x}$

4) $\operatorname{csch} x = \frac{1}{\sinh x}$

5) $\operatorname{sech} x = \frac{1}{\cosh x}$

6) $\operatorname{csch} x = \frac{1}{\tanh x}$

Note

$\sinh 0 = 0$

$\cosh 0 = 1$

Note $\cosh^2 x - \sinh^2 x = 1$

Q₁ Find Value of cosh0

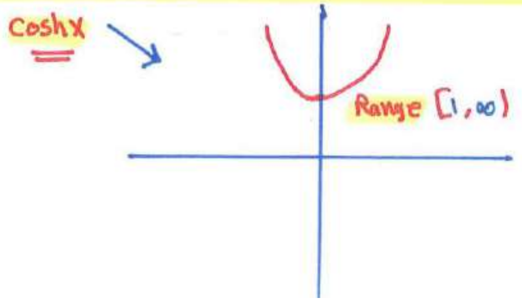
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

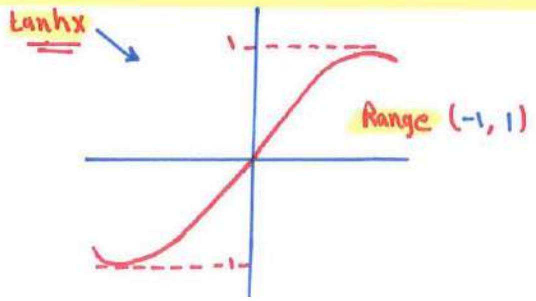
$$= \frac{e^0 + e^0}{2} = \frac{2}{2} = 1$$

Q₂ find Value of sinh(ln2)

كيف اجت $\rightarrow e^{-\ln 2} = e^{\ln 2^{-1}} = 2^{-1} = \frac{1}{2^1}$

$$\sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{1.5}{2}$$





Q_3 $\cosh x = \frac{5}{3}$, $x > 0$ find $\coth x$.

في سؤال طلب coth x بس احنا جبنا كل اشي 🙂 .

1) $\operatorname{sech} x = \frac{3}{5}$

2) $\cosh^2 x - \sinh^2 x = 1$

$$\left(\frac{5}{3}\right)^2 - \sinh^2 x = 1$$

$$\sinh^2 x = \frac{25}{9} - \frac{9}{9} \Longrightarrow \frac{16}{9} = \sinh^2 x$$

→ توحيد مقام

$$\therefore \sinh x = \pm \frac{4}{3}$$

$$\therefore \sinh x = \boxed{\frac{4}{3}}$$

ملاحظة : إذا كانت x أكبر من (0) تكون الـ (sinh) موجبة , (tanh) موجبة .

ملاحظة : إذا كانت x أقل من (0) تكون الـ (sinh) سالبة , (tanh) سالبة .

ملاحظة : الـ (cosh) دائماً موجبة .

3) $\operatorname{csch} x = \frac{3}{4}$

4) $\tanh x = \frac{\sinh}{\cosh} = \frac{\frac{4}{\cancel{3}}}{\frac{5}{\cancel{3}}} = \frac{4}{5}$

5) $\coth x = \frac{5}{4}$

Q_4 $\operatorname{sech} x = \frac{1}{5}$, $x < 0$, find $\sinh x$.

$\cosh x = 5 \Longrightarrow$

$$\cosh^2 x - \sinh^2 x = 1$$
$$25 - \sinh^2 x = 1$$
$$\sinh^2 x = 25 - 1$$
$$\sinh^2 x = 24$$
$$\sinh x = \pm\sqrt{24}$$
$$\sinh x = -\sqrt{24} \quad , \boxed{x < 0}$$

Note $1 - \tanh^2 x = \operatorname{sech}^2 x$

$F(x)$		$f'(x)$
$\sinh x$	$\longrightarrow$	$\cosh x$.
$\cosh x$	$\longrightarrow$	$\sinh x$.
$\tanh x$	$\longrightarrow$	$\operatorname{sech}^2 x$.
$\coth x$	$\longrightarrow$	$-\operatorname{csch}^2 x$.
$\operatorname{sech} x$	$\longrightarrow$	$-\operatorname{sech} x . \tanh x$.
$\operatorname{csch} x$	$\longrightarrow$	$-\operatorname{csch} x . \coth x$.

$\sinh 0 = 0$
$\cosh 0 = 0$

Q_1 $y = \operatorname{sech}\sqrt{x}$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\operatorname{sech} x . \tanh x . \frac{1}{2\sqrt{x}}$$

Q_2 $F(x) = e^x \cosh x$, Find $f'(0)$

$$f'(x) = e^x \overset{0}{\cancel{\sinh x}} + \cosh x . e^x = 0 + (1).(1) = 1$$

Q_3 $\lim\limits_{x \to \infty} \cosh x$

$$= \lim_{x \to \infty} \frac{e^x + e^{-x}}{2} = \frac{\infty + 0}{2} = \frac{\infty}{2} = \infty$$

Q_4 $\lim\limits_{x \to \infty} \frac{\sinh x}{e^x}$

$$= \lim_{x \to \infty} \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x}{1}} \longrightarrow \lim_{x \to \infty} \frac{e^x - e^{-x}}{2} . \frac{1}{e^x}$$

$$= \lim_{x \to \infty} \frac{e^x - e^{-x}}{2e^x} = \lim_{x \to \infty} \frac{e^x}{2e^x} - \frac{e^{-x}}{2e^x})$$

$$= \lim_{x \to \infty} \frac{1}{2} - \frac{1}{2e^{2x}} = \frac{1}{2} - 0 = \frac{1}{2}$$

Q1 $y = \log_3 3^{2x} \sqrt[5]{x^2-1}$, find $\frac{dy}{dx}$

Q2 find $\frac{d}{dx}\left(\log_7 7^{3x} \sqrt[3]{x^2-1}\right)$

Q3 find $\lim\limits_{x\to-\infty} \sin^{-1}\left(\frac{1}{1+e^x}\right) =$

Q4 The line $y=x+b$ is tangent to the curve $y=e^x$ then $b=$

Q5 $f(x) = \ln x + 2x + 1$, then $(f^{-1})'(3) =$

Q6 $f(x)$ is cts function, $f(1)=2$, $f(3)=3$, using IVT $f(x)=5$ must have at least two solutions in the interval $(1,3)$ if

a. $2 \leq f(2) \leq 3$

b. $3 \leq f(2) < 5$

c. $f(2) \leq 2$

d. $f(2) < 2$

e. $f(2) > 5$

Q7 $\lim\limits_{x\to-\infty} \frac{1+x}{1-3|x|} =$

Q8 $f(3x-1) = 9x^2+3x$, find $f'(2)$

Q9 Use linear approximation to find $\tan^{-1} 1.05$

Q10 the slope of the normal line of the curve $x^2+y^2=5x$ at $(1,2)$ equals

Q11 the curve $y=\tan^{-1} x$ has tangent line parallel to the line $y = 1+\frac{1}{2}x$ when $x=$

Q12 $y = x^{\ln x}$, find $\frac{dy}{dx}$

Q13 $\lim\limits_{x\to0} \frac{x}{4^x-1}$

Q14 let $f(x) = \begin{cases} ax+1, & x<1 \\ bx^2, & x\geq1 \end{cases}$, if $f(x)$ is diff at $x=1$, then $a=$

Q15 $\tanh x = 0.5$, $\sinh x =$

Q16 $\lim\limits_{x\to\infty} \frac{\tanh x}{e^x}$ equals

Q17 $y = x^{\cos x}$, find $\frac{dy}{dx}$

Q18 $y = \cos^{-}(\cos x)$, find $\frac{dy}{dx}$

Q19 find $\frac{d}{dx}(x \sin \sqrt{x})$

Q20 the equation $x^3 + 3x = 2$ has solution in

a- $[1,3]$ b- $[1,2]$ c- $[2,3]$ d- $[0,1]$

Q21 $f(x) = \sin^2(4x+1)$, find $f'(x)$

Q22 $g(x) = f(2x)$, $f'(10) = 6$, then find $g'(5)$

Q23 $\frac{d}{dx} f(3x) = x^2$, find $f'(x)$

Q24 $f(x) = |6x+3|$ not diff at $x =$

Q25 $y = f(x^2+1)$, $f'(2) = 2$, $f(2) = 3$, $\frac{dy}{dx}\Big|_{x=1}$

Q26 $\lim\limits_{x\to\infty} \ln\sqrt{4x^2+x+1} - \ln(x+1)$

Q27 $\lim\limits_{x\to\infty} \tanh 4x$

Q28 $f(x) = \begin{cases} ax+2 & , x \geq 1 \\ bx^2+3x & , x < 1 \end{cases}$ $f(x)$ is diff at $x=1$, find a, b

Q_{29} $\tanh x = \frac{-1}{2}$, find sech x

Q_{30} $f(x) = (x-2)^{\frac{2}{3}}$, find vertical tangent

Q_{31} $f(x) = (\sin x)^{\frac{1}{x}}$

Q_{32} $\lim\limits_{x \to \infty} \frac{\sin^2 x}{2^x \ln x}$

Q_{33} $f(x) = \frac{\sqrt{x^2+x}}{3x-2}$, find h.a

Q_{34} the slope of the line tangent to the curve $x^2+xy+y^2=1$ at the point $(1,y)$ is (1), find y

Q_{35} the slope of the tangent to the curve $x^2+xy+y^2=3$ at $(-1,-1)$ is

Q_{36} $f(x) = |x(x^2-2x)|$ not diff at =

Q_{37} $f(x) = 5^{\tan 4x}$, then $f'(x) =$

Q_{38} $f(x) = \cos \frac{1}{x^2}$, $f'(x) =$

Q_{39} $y = \sinh \sqrt{x}$, find $\frac{dy}{dx}$

Q_{40} $\lim\limits_{x \to \infty} \frac{\cosh x}{e^x}$