MAESTRO

CALCULUS 1

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* Hyperbolic Function :

1) $\sinh x = \dfrac{e^{x} - e^{-x}}{2} \longrightarrow$ odd

2) $\cosh x = \dfrac{e^{x} + e^{-x}}{2} \longrightarrow$ even

3) $\tanh x = \dfrac{\sinh x}{\cosh x}$

4) $\operatorname{csch} x = \dfrac{1}{\sinh x}$

5) $\operatorname{sech} x = \dfrac{1}{\cosh x}$

6) $\operatorname{csch} x = \dfrac{1}{\tanh x}$

Note

$\sinh 0 = 0$

$\cosh 0 = 1$

Note $\cosh^2 x - \sinh^2 x = 1$

Q1 Find Value of cosh0

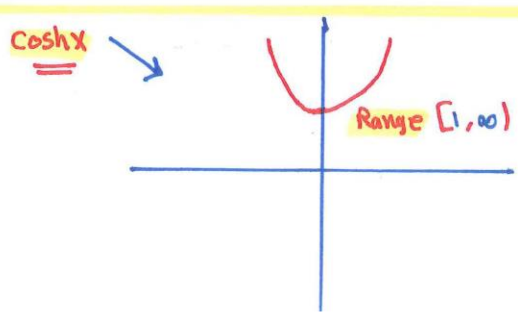
$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

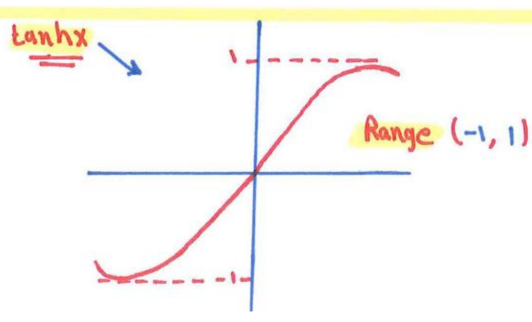
$$= \frac{e^{0} + e^{0}}{2} = \frac{2}{2} = 1$$

Q2 find Value of sinh(ln2)

$$\sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{1.5}{2}$$

كيف اجت $e^{-\ln 2} = e^{\ln 2^{-1}} = 2^{-1} = \frac{1}{2^1}$





Q_3 $\cosh x = \frac{5}{3}$, $x > 0$ find $\coth x$.

في سؤال طلب $\coth x$ بس احنا جبنا كل اشي .

1) $\operatorname{sech} x = \frac{3}{5}$

2) $\cosh^2 x - \sinh^2 x = 1$

$\left(\frac{5}{3}\right)^2 - \sinh^2 x = 1$

$\sinh^2 x = \frac{25}{9} - \frac{9}{9} \Longrightarrow \frac{16}{9} = \sinh^2 x$

توحيد مقام

$\therefore \sinh x = \pm \frac{4}{3}$

$\therefore \sinh x = \boxed{\frac{4}{3}}$

ملاحظة : إذا كانت x أكبر من (0) تكون الـ (sinh) موجبة , (tanh) موجبة .

ملاحظة : إذا كانت x أقل من (0) تكون الـ (sinh) سالبة , (tanh) سالبة .

ملاحظة : الـ (cosh) دائماً موجبة .

3) $\operatorname{csch} x = \frac{3}{4}$

4) $\tanh x = \frac{\sinh}{\cosh} = \frac{\frac{4}{\cancel{3}}}{\frac{5}{\cancel{3}}} = \frac{4}{5}$

5) $\coth x = \frac{5}{4}$

Q_4 $\operatorname{sech} x = \frac{1}{5}$, $x < 0$, Find $\sinh x$.

$\cosh x = 5 \Longrightarrow$

$\cosh^2 x - \sinh^2 x = 1$

$25 - \sinh^2 x = 1$

$\sinh^2 x = 25 - 1$

$\sinh^2 x = 24$

$\sinh x = \pm\sqrt{24}$

$\sinh x = -\sqrt{24}$, $\boxed{x < 0}$

Note $1 - \tanh^2 x = \operatorname{sech}^2 x$

$F(x)$		$F'(x)$
$\sinh x$	$\longrightarrow$	$\cosh x$.
$\cosh x$	$\longrightarrow$	$\sinh x$.
$\tanh x$	$\longrightarrow$	$\operatorname{sech}^2 x$.
$\coth x$	$\longrightarrow$	$-\operatorname{csch}^2 x$.
$\operatorname{sech} x$	$\longrightarrow$	$-\operatorname{sech} x \cdot \tanh x$.
$\operatorname{csch} x$	$\longrightarrow$	$-\operatorname{csch} x \cdot \coth x$.

$\sinh 0 = 0$
$\cosh 0 = 0$

Q_1 $y = \operatorname{sech}\sqrt{x}$, Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\operatorname{sech} x \cdot \tanh x \cdot \frac{1}{2\sqrt{x}}$$

Q_2 $F(x) = e^x \cosh x$, Find $F'(0)$

$$F'(x) = e^x \cancel{\sinh x}^{\,0} + \cosh x \cdot e^x = 0 + (1)\cdot(1) = 1$$

Q_3 $\lim_{x \to \infty} \cosh x$

$$= \lim_{x \to \infty} \frac{e^x + e^{-x}}{2} = \frac{\infty + 0}{2} = \frac{\infty}{2} = \infty$$

Q_4 $\lim_{x \to \infty} \frac{\sinh x}{e^x}$

$$= \lim_{x \to \infty} \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x}{1}} \longrightarrow \lim_{x \to \infty} \frac{e^x - e^{-x}}{2} \cdot \frac{1}{e^x}$$

$$= \lim_{x \to \infty} \frac{e^x - e^{-x}}{2e^x} = \lim_{x \to \infty} \frac{e^x}{2e^x} - \frac{e^{-x}}{2e^x}$$

$$= \lim_{x \to \infty} \frac{1}{2} - \frac{1}{2e^{2x}} = \frac{1}{2} - 0 = \frac{1}{2}$$

Q1 $y = \log_3 3^{2x} \sqrt[5]{x^2-1}$, find $\frac{dy}{dx}$

Q2 find $\frac{d}{dx}\left(\log_7 7^{3x} \sqrt[3]{x^2-1}\right)$

Q3 find $\lim_{x \to -\infty} \sin^{-1}\left(\frac{1}{1+e^x}\right) =$

Q4 The line $y = x + b$ is tangent to the curve $y = e^x$ then $b =$

Q5 $f(x) = \ln x + 2x + 1$, then $(f^{-1})'(3) =$

Q6 $f(x)$ is cts function, $f(1) = 2$, $f(3) = 3$, using IVT $f(x) = 5$ must have at least two solutions in the interval $(1,3)$ if

a- $2 \leq f(2) \leq 3$
b- $3 \leq f(2) < 5$
c- $f(2) \leq 2$
d- $f(2) < 2$
e- $f(2) > 5$

Q7 $\lim_{x \to -\infty} \frac{1+x}{1-3|x|} =$

Q8 $f(3x-1) = 9x^2 + 3x$, find $f'(2)$

Q9 Use linear approximation to find $\tan^{-1} 1.05$

Q10 the slope of the normal line of the curve $x^2 + y^2 = 5x$ at $(1,2)$ equals

Q11 the curve $y = \tan^{-1} x$ has tangent line parallel to the line $y = 1 + \frac{1}{2}x$ when $x =$

Q12 $y = x^{\ln y}$, find $\frac{dy}{dx}$

Q13 $\lim_{x \to 0} \frac{x}{4^x - 1}$

Q14 let $f(x) = \begin{cases} ax + 1 , & x < 1 \\ bx^2 , & x \geq 1 \end{cases}$, if $f(x)$ is diff at $x = 1$, then $a =$

Q15 $\tanh x = 0.5$, $\sinh x =$

Q16 $\lim_{x\to\infty} \frac{\tanh x}{e^x}$ equals

Q17 $y = x^{\cos x}$, find $\frac{dy}{dx}$

Q18 $y = \cos^{-1}(\cos x)$, find $\frac{dy}{dx}$

Q19 find $\frac{d}{dx}(x\sin^{-1}\sqrt{x})$

Q20 the equation $x^3+3x=2$ has solution in
a- $[1,3]$ b- $[1,2]$ c- $[2,3]$ d- $[0,1]$

Q21 $f(x) = \sin^2(4x+1)$, find $f'(x)$

Q22 $g(x) = f(2x)$, $f'(10)=6$, then find $g'(5)$

Q23 $\frac{d}{dx} f(3x) = x^2$, find $f'(x)$

Q24 $f(x) = |6x+3|$ not diff at $x=$

Q25 $y = f(x^2+1)$, $f'(2)=2$, $f(2)=3$, $\left.\frac{dy}{dx}\right|_{x=1}$

Q26 $\lim_{x\to\infty} \ln\sqrt{4x^2+x+1} - \ln(x+1)$

Q27 $\lim_{x\to\infty} \tanh 4x$

Q28 $f(x) = \begin{cases} ax+2 & , x\geq 1 \\ bx^2+3x & , x<1 \end{cases}$ f(x) is diff at $x=1$, find a, b

Q_{29} $\tanh x = \frac{-1}{2}$, find sechx

Q_{30} $f(x) = (x-2)^{\frac{2}{3}}$, find vertical tangent

Q_{31} $f(x) = (\sin x)^{\frac{1}{x}}$

Q_{32} $\lim_{x \to \infty} \frac{\sin^2 x}{2^x \ln x}$

Q_{33} $f(x) = \frac{\sqrt{x^2+x}}{3x-2}$, find h.a

Q_{34} the slope of the line tangent to the curve $x^2+xy+y^2=1$ at the point $(1,y)$ is (1), find y

Q_{35} the slope of the tangent to the curve $x^2+xy+y^2=3$ at $(-1,-1)$ is

Q_{36} $f(x) = |x(x^2-2x)|$ not diff at =

Q_{37} $f(x) = 5^{\tan 4x}$, then $f'(x) =$

Q_{38} $f(x) = \cos\frac{1}{x^2}$, $f'(x) =$

Q_{39} $y = \sinh\sqrt{x}$, find $\frac{dy}{dx}$

Q_{40} $\lim_{x \to \infty} \frac{\cosh x}{e^x}$

Ch. 4.2

* Mean value Theorm (MVT)

$$f'(x) = \frac{F(b)-F(a)}{b-a} \quad , [a,b] \rightarrow \text{given}$$

Q_1 $F(x) = x^3 + x - 1$, $[\overset{a}{1}, \overset{b}{3}]$, Find x that satisfies MVT ?

$$f'(x) = 3x^2 + 1 = \frac{F(b)-F(a)}{b-a}$$

$$3x^2 + 1 = \frac{F(3)-F(1)}{3-1} \Rightarrow 3x^2 + 1 = \frac{29-1}{2}$$

$$3x^2 + 1 = 14 \Rightarrow \frac{\cancel{3}x^2}{\cancel{3}} = \frac{13}{3}$$

$$\therefore x = \pm\sqrt{\frac{13}{3}}$$

تهمل لأنها خارج الفترة .

$$\therefore x = \sqrt{\frac{13}{3}}$$

ملاحظة : يجب أن تكون قيمة x داخل الفترة المعطاة في السؤال .

ملاحظة : لو كانت x = 1 تهمل حتى لو كان في مساواة .

ملاحظة : الاطراف تهمل حتى لو كانت الفترة مغلقه .

* Rolle's Theorm $\longrightarrow f'(x) = 0$

Q_2 $x^3 - 3x + 1$, $[-1, 2]$. Find x that satisfies Rolle's Theorm .

$$f'(x) = 0$$

$$f'(x) = 3x^2 - 3 = 0$$

$$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{3}{3} \Rightarrow x^2 = 1 \longrightarrow x = \pm 1$$

تهمل (لأنها طرف) .

مهم جداً **Q_3** $f'(x) \leqslant 5$, $f(0) = -3$, how large can $F(2)$ possible be?

أو how small → it's a key word For (MVT)

$$f'(x) = \frac{F(b) - F(a)}{b - a}$$

$$5 = \frac{F(2) - F(0)}{2 - 0} \longrightarrow 5_{\times 2} = \frac{F(2) + 3}{2} \times 2$$

$$10 = F(2) + 3$$

$$7 = F(2) \Longrightarrow F(2) \leqslant 7$$

مهم جداً **Q_4** $f(1) = 10$, $f'(x) \geqslant 2$, how small can $F(4)$ be??

$$f'(x) = \frac{F(b) - F(a)}{b - a}$$

$$2 = \frac{f(4) - f(1)}{4 - 1} \Longrightarrow 2_{\times 3} = \frac{F(4) - 10}{3} \times 3$$

$$6 = F(4) - 10$$

$$16 = F(4) \Longrightarrow F(4) \geqslant 16$$

Q_5 Does there exist a Function such that $F(0) = -1$, $F(2) = 4$, $F'(x) \leqslant 2$ $\underset{\text{For all}}{\forall}$ value(s) of x ?

$$f'(x) = \frac{F(4) - F(0)}{4 - 0}$$

$$f'(x) = \frac{4+1}{4} = \frac{5}{2} = 2.5 \quad \therefore \text{No}$$ لأنها أكبر من 2

verey important Qs

Note $\longrightarrow$ $f'(x) = 0 \xrightarrow{\text{لأنو}} F(b) = F(a)$

Q1 $F(x) = x^2 + 3x$, $[1, 3]$ satisfies Roll's Th'm

$F(1) = 1 + 3 = 4$

$F(3) = 18 \quad \therefore F(a) \neq F(b) \quad \therefore \boxed{\text{False}}$

Q2 $F(x) = x^3 + x + 1$, $[a, b]$, satisfies Roll's Th'm

a) $F(a) > F(b)$ b) $F(a) < F(b)$ c) $F(a) = F(b)$

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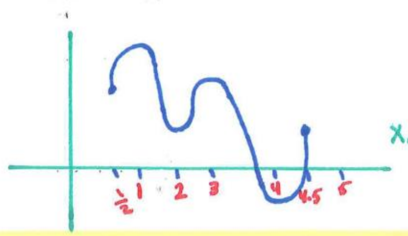
ch. 4.1 Maximum and Minimum ((قيم y القصوى))

abs max (extrem)
أكبر قيمة لـ y موجودة في السؤال .
F(1)

abs min (extrem)
أقل قيمة لـ y موجودة في السؤال .
F(4)

local max (relative)
هي نقطة أكبر مما حولها فقط ويكون شكلها على شكل قمة .
F(1) , F(3)

local min (relative)
هي نقطة أقل مما حولها فقط ويكون شكلها على شكل قاع .
F(2), F(4)



ملحوظة ← الأطراف لاتعتبر (local) .

ملحوظة ← أي نقطة تحتوي على min أو max تسمى نقطة حرجة قيم x (critical number) ← (C)

ملحوظة ← الأطراف المغلقة أيضاً تعتبر (critical number)

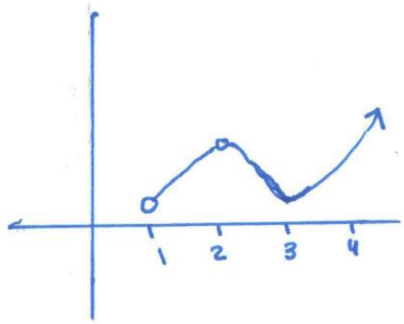
Q1 Find critical number

$x = \underbrace{1, 2, 3, 4}_{\text{قمم max وقاع min}} , \underbrace{\tfrac{1}{2}, 6}_{\text{أطراف مغلقة}}$

* critical number $\longrightarrow$ قيم x

* critical points $\longrightarrow$ (y, x) ← (x وصورتها)

Q_2



a) Find abs max $\longrightarrow$ no abs max $\longrightarrow \infty$ ← لأن الرسمة غير منتهية

b) Find critical numbers $\longrightarrow$ $x = 3$

Q_3 How to Find max and min From $y = F(x)$ $\longrightarrow$ (من خلال الاقتران)

a) Find Domain $\longrightarrow$ بالغالب معطاة

b) find cirtical number(s)

$f'(x) = 0$ — الأطراف المغلقة

بشتق الاقتران وبساويه بالصفر
وبسميها أطراف حرجة بشرط
أن تكون داخل ال Domain.

Q_4 $F(x) = 2x \ln x$, Find critical numbers.

$$D = (0, \infty)$$

$$f'(x) = 0 \implies 2\cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2 = 0$$

$$2 + 2\ln x = 0 \implies \frac{2\ln x}{2} = \frac{-2}{2}$$

$$\therefore e^{\ln x} = e^{-1} \implies \therefore x = e^{-1} = \frac{1}{e}$$

نقطة حرجة

Q_5 $F(x) = x^3 - 3x^2 + 1$, $[-\frac{1}{2}, 4]$ Find (c.n)

$\boxed{x = -\frac{1}{2}}$, $\boxed{x = 4}$ c.n $\longrightarrow$ لأنهم أطراف مغلقة

$F'(x) = 3x^2 - 6x = 0$

$3x^2 - 6x = 0 \implies 3x(x - 3) = 0$

$3x = 0$, $x - 3 = 0$

$\therefore \boxed{x = 0}$ $\boxed{x = 3}$

$\rightarrow$ (c.n) لأنهم داخل الDomain

ملحوظة ← نجد صور كل (c.n) ، أكبر صورة هي $\left(\begin{matrix}abs\\max\end{matrix}\right)$ وأصغر صورة $\left(\begin{matrix}abs\\min\end{matrix}\right)$

$\rightarrow F(-\frac{1}{2}) = -\frac{1}{8}$

$F(4) = 27 \longrightarrow$ (abs max)

$F(0) = 1$

$F(2) = -3 \longrightarrow$ (abs min)

Q_6 $F(x) = 2x^3 - 3x^2 - 36x$, Find c.n

$D = \mathbb{R} \rightarrow (-\infty, \infty)$

$F'(x) = \frac{6x^2}{6} - \frac{6x}{6} - \frac{36}{6} = 0$

$= x^2 - x - 6 = 0$

$= x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$\boxed{x = 3}$ $\boxed{x = -2}$

c.n $\longrightarrow$ لأنهم داخل الDomain

Q_7 $F(x) = x + \frac{1}{x}$, Find c.n

$D = \mathbb{R} - \{0\}$

$F'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

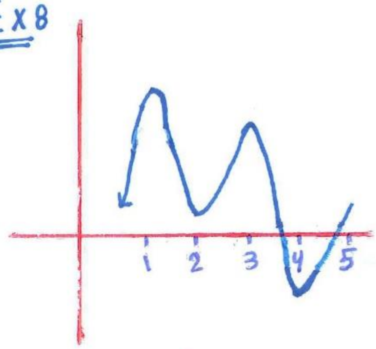
وحدنا المقام $\rightarrow \frac{x^2 - 1}{x^2} = 0$

$x^2 - 1 = 0 \longrightarrow \boxed{x = 1}$, $\boxed{x = -1}$

$x^2 = 0 \longrightarrow \boxed{x = 0}$

ملاحظة: إذا كانت المشتقة مكونة من عدة كسور، يجب توحيد المقامات قبل المساواة بالصفر.

Ex 8



abs max = $f(1)$

abs min = no abs min

$F(x)$ increasing $\longrightarrow (-\infty, 1), (2,3), (4,5)$ دائماً الفترات مفتوحة ← *

$F(x)$ decreasing $\longrightarrow (1,2), (3,4)$

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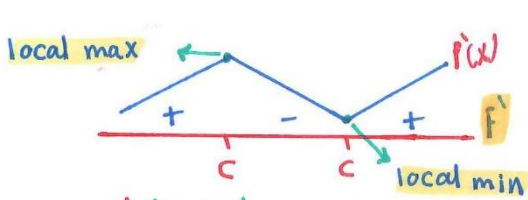
* → important   How to Find max and min From $y = F(x)$

1) find Domain.
2) Find critical number(s).
3) increasing and decreasing.



$\Longrightarrow$

(المشتقة)   (الاقتران)

$f'(x) \overset{+}{>} 0$, $F(x)$ inc ↗

$f'(x) \overset{-}{<} 0$, $F(x)$ dec ↘

4) local max and min.

Q1 $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$, Find local max and min.

① Domain $\longrightarrow$ $D = \mathbb{R}$

② $f'(x) = 0$

$= 12x^3 - 12x^2 - 24x = 0$

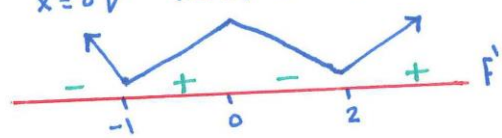
$= (12x)(x^2 - x - 2) = 0$

$= (12x)(x-2)(x+1) = 0$

$x = 0$ ✓ $x = 2$ ✓ $x = -1$ ✓

Note

أي رقم بطلع لازم احطه على خط الاعداد.

③ 

dec $\longrightarrow$ $(-\infty, -1)$, $(0, 2)$

inc $\longrightarrow$ $(-1, 0)$, $(2, \infty)$

④ local max = $f(0)$ at $x = 0$

local min = $F(-1)$ at $x = -1$

local min = $F(2)$ at $x = 2$

Q2 $F(x) = x + \frac{1}{x}$, Find max and min local.

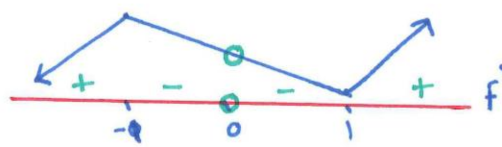
① Domain $\longrightarrow$ $R - \{0\}$

② $f'(x) = 1 + \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = 0$

$x^2 - 1 = 0$ → $x = 1$ ✓ , $x = -1$ ✓

$x^2 = 0 \longrightarrow x = 0$ ✗ not critical

* بس لازم كل الأرقام أحطهم على خط الاعداد.

③ 

F inc $(-\infty, -1)$, $(1, \infty)$

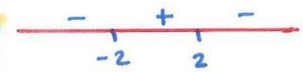
F dec $(-1, 0)$, $(0, 1)$

④ local max = $F(-1)$

local min = $F(1)$

Q_3 $f(x) = \ln(4-x^2)$

① Domain $\longrightarrow$ $D = (-2, 2)$ الفترة الموجبة

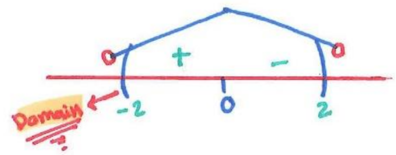


② $f'(x) = \dfrac{-2x}{4-x^2} = 0$

$\rightarrow -2x = 0 \longrightarrow x = 0$ ✓

$\rightarrow 4-x^2 = 0 \longrightarrow x = 2$ ✗ not critical, $x = -2$ ✗

③



$\rightarrow$ f inc $(-2, 0)$

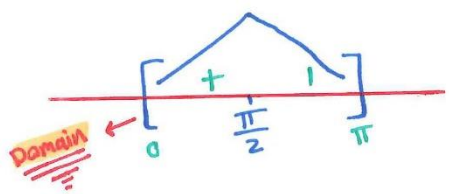
$\rightarrow$ f dec $(0, 2)$

④ local max $= f(0) = \ln 4$

Q_4 $f(x) = \sin x$, $[0, \pi]$ ① Domain

② $f'(x) = \cos x = 0 \longrightarrow x = \frac{\pi}{2}$ ✓

③



$\rightarrow$ f inc $(0, \frac{\pi}{2})$

$\rightarrow$ f dec $(\frac{\pi}{2}, \pi)$

④ local max $= f(\frac{\pi}{2}) = 1$

$D = \mathbb{R}$

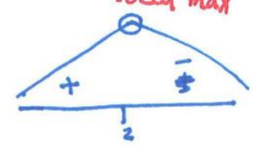
Q5 suppose $F(x)$ is cts on $(-\infty, \infty)$ and $F'(2) = 0$

then $x = 2$ is a critical number

Q6 $F(x)$ is cts $(-\infty, \infty)$, $F'(2) = 0$, $F'(1) \overset{+}{>} 0$, $F'(3) \overset{-}{<} 0$

then :

local max



a) $F(x)$ has local min at $x = 2$

$\boxed{b)}$ $F(x)$ has local max at $x = 2$

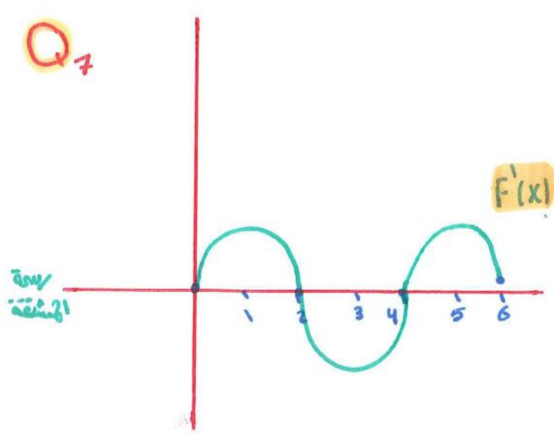
c) $F(x)$ is decreasing $(-\infty, 2)$

d) $F(x)$ is increasing $(2, \infty)$

~~x~~ → first derivative

Rules →
1) ∩ corcave down
2) ∪ corcave up
3) f' +ve → $f \uparrow$
4) f' -ve → $f \searrow$
5) $f' \uparrow$ → f ∪
6) $f' \searrow$ → f ∩

Q7



answer 80

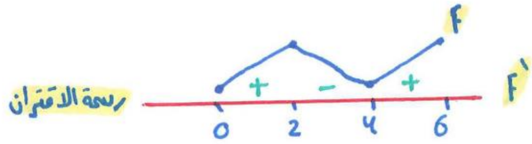
① f is inc at (f'+ve)

$(0, 2), (4, 6)$

② F is dec at (F'-ve)

$(2, 4)$

③ local max at $x = 2$ → inc, dec لازم اطلع



④ critical number(s)

0, 6 → ends , 2, 4

⑤ F concave up → F' ↑

(0,1) , (3,5)

⑥ F concave down → F' ↓

(1,3) , (5,6)

⑦ at (2,3) F(x) is

a) increasing and conacave down.

b) increasing and concave up.

c) decreasing and concave up.

d) decreasing and concave down.

⑧ at (4,6) F(x)

a) increasing and concave up.

b) decresing and coreave up.

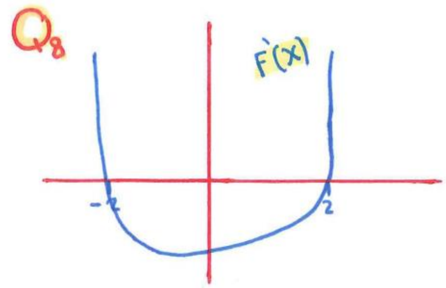
c) increasing and concave down

d) increasing

⑨ at (1,2) F(x) is

F increasing and concave down.

Q_8



answer → in the interval $(2, \infty)$

F(x) is → F increasing and concave up.

review ::

f' +ve $\longrightarrow$ $F(x) \nearrow$ f' -ve $\longrightarrow$ $F(x) \searrow$	— U, ∩ f — ↗, ↘ f', f — +ve, -ve f'
$f' \nearrow$ $\longrightarrow$ $f(x)$ U $f' \searrow$ $\longrightarrow$ $F(x)$ ∩	

concavity :: (How to find it)

1) Find domain.

2) $f''(x) = 0$, (p) $\longrightarrow$ inflection number(s) $\longrightarrow$ ولازم :
① يكون داخل ال Domain
② change in concavity

$$\begin{array}{ccccc} + & & - & & + \\ \hline & p & & p & \end{array} \quad f''$$

so → إذا f'' +ve $\longrightarrow$ f U
f'' -ve $\longrightarrow$ f ∩ $\longrightarrow$ — U, ∩ f
— +ve, -ve f''

Q, $f(x) = x^4 - 4x^3$, Find the interval of concavity and inflection point.

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$12x = 0$, $x - 2 = 0$

$\boxed{x = 0}$ $\boxed{x = 2}$

$$\begin{array}{ccccc} \text{U} & & \cap & & \text{U} \\ + & & - & & + \\ \hline & 0 & & 2 & \end{array} \quad f''$$

① $\left.\begin{array}{l} x = 0 \\ x = 2 \end{array}\right\}$ inf. number

② U $\longrightarrow$ $(-\infty, 0)$, $(2, \infty)$ ∩ $\longrightarrow$ $(0, 2)$

second derivative test

:كيفية ايجاد الـ max , min من المشتقة الثانية

1) find critical number(s).

2) $f''(c)$ → نعوض الـ critical number داخل المشتقة الثانية

- $f''(c) > 0$ local min ∪
- $f''(c) < 0$ local max ∩
- $f''(c) = 0$ horizontal tangent

very important Q 80

Q_2 $f(x)$ is cts at $(-\infty,\infty)$ and $f'(2) = 0$, $f''(2) = -3$, then $F(x)$ at $x = 2$ has ...

critical num

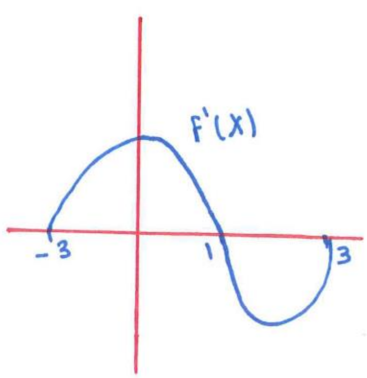
∩

1) local max .
2) local min .
3) horizontal tangent
4) abs max .

Q_3 $F(x)$ is cts at $\mathbb{R}$, $F(3) = 2$, $f'(3) = 0$, $F''(3) = 0$, then $F(x)$ at $x = 3$ has ...

1) local max .
2) local min .
3) horizontal tangent .
4) abs min .

Q_4



at $(-3,0)$ $F(X)$ is

concave up and increasing

$f'(x) \nearrow \longrightarrow F(X)$ ∪

$f'(x)$ +ve $\longrightarrow F(X) \nearrow$

Slant asymptote (Oblique)

هو ناتج القسمة الطويلة لاقتران كثير حدود ع
اقتران كثير حدود آخر شرط ان تكون قوة البسط اكبر من
قوة المقام بمقدار 1 فقط

$$f(x) = \frac{g(x)}{h(x)} \quad \begin{matrix} \to \text{Poly} \\ \to \text{Poly} \end{matrix}$$

Q₁ find slant asymptote for $f(x) = \dfrac{x^2+1}{x-1}$

$$\begin{array}{r|l} & x+1 \\ \hline x-1 & x^2+1 \\ & \cancel{-x^2}+x \\ \hline & x+1 \\ & -x+1 \\ \hline & 2 \end{array}$$

slant asy $\boxed{y = x+1}$

Q₂ find slant asymptote for $f(x) = \begin{cases} \dfrac{x^2}{x-1}, & x > 1 \\ \dfrac{x^2}{x-2}, & x \leq 1 \end{cases}$

$$\begin{array}{r|l} & x+1 \\ \hline x-1 & x^2 \\ & -x^2+x \\ \hline & x \\ & x+1 \\ \hline \end{array} \to y = x+1$$

$$\begin{array}{r|l} & x+2 \\ \hline x-2 & x^2 \\ & -x^2+2x \\ \hline & 2x \end{array} \to y = x+2$$

so we have two slant asy

$y = x+1$

$y = x+2$

L'hopital Rule

→ Case 1 اذا كان ناتج التعويض المباشر $\frac{0}{0}$, $\frac{\infty}{\infty}$

الحل : نشتق البسط ونشتق المقام ثم نعوض مرة اخرى

Q_1 $\lim\limits_{x\to 2} \frac{x^2-4}{x-2} \nearrow \frac{0}{0}! = \lim\limits_{x\to 2} \frac{2x}{1} = 4$

Q_2 $\lim\limits_{x\to 0} \frac{3^x-1}{x} \nearrow \frac{0}{0}! = \lim\limits_{x\to 0} \frac{3^x . \ln 3}{1} = \ln 3$

Q_3 $\lim\limits_{x\to \infty} \frac{x^2}{e^x} \nearrow \frac{\infty}{\infty}! = \lim\limits_{x\to \infty} \frac{2x}{e^x} \nearrow$ لازالت $\frac{\infty}{\infty}!$ $= \lim\limits_{x\to \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$

نشتق مرة اخرى

Q_4 $\lim\limits_{x\to \infty} \frac{\ln x}{\sqrt{x}} \nearrow \frac{\infty}{\infty}! = \lim\limits_{x\to \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim\limits_{x\to \infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{1} = 0$

لأن البسط اصغر من المقام او بالامكان لوبيتال مرة اخرى

Q_5 $\lim\limits_{x\to 0} \frac{e^{2x}-1-2x}{x^2} \nearrow \frac{0}{0}! = \lim\limits_{x\to 0} \frac{e^{2x} . 2 - 2}{2x} \nearrow \frac{0}{0}! = \lim\limits_{x\to 0} \frac{\cancel{2} e^{2x} . 2}{\cancel{2}} = 2$

→ Case 2 اذا كان ناتج التعويض المباشر $0(\pm\infty)$

الحل : نقوم بانزال احد الاقترانات للمقام ثم نشتق

Q_1 $\lim\limits_{x\to 0^+} x \ln x \nearrow 0.(-\infty)! = \lim\limits_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{-1} = \frac{0}{-1} = 0$

Q_2 $\lim\limits_{x\to 0^+} \cot x . \ln(4x+1) = \lim\limits_{x\to 0^+} \frac{\ln(4x+1)}{\tan x} = \lim \frac{\frac{4}{4x+1}}{\frac{\sec^2 x}{1}}$

$= \lim \frac{4}{4x+1} \cdot \frac{1}{\sec^2 x} = \frac{4}{1} \cdot \frac{1}{1} = 4$

→ case 3 ← اذا احتوى السؤال على كسرين, نوحد المقامات ثم نستخدم قاعدة لوبيتال

Q_1 $\lim_{x\to 0^+} \frac{4}{x} - \frac{4}{e^x - 1}$ نوحد المقامات اولاً

$= \lim_{x\to 0^+} \frac{4e^x - 4 - 4x}{xe^x - x} \to \frac{0}{0}! = \lim_{x\to 0^+} \frac{4e^x - 4}{xe^x + e^x - 1} \to \frac{0}{0}! = \lim_{x\to 0^+} \frac{4e^x}{xe^x + e^x + e^x} = \frac{4}{2} = 2$

Q_2 $\lim_{x\to 1^+} \left(\frac{1}{\ln x} - \frac{x}{x-1}\right)$ نوحد المقامات

انتبه للاشارات!

$= \lim_{x\to 1^+} \frac{x - 1 - x\ln x}{x\ln x - \ln x} \to \frac{0}{0}! = \lim_{x\to 1^+} \frac{1 - x\cdot\frac{1}{x} - \ln x}{x\cdot\frac{1}{x} + \ln x - \frac{1}{x}} = \lim_{x\to 1^+} \frac{-\ln x}{1 + \ln x - \frac{1}{x}} \to \frac{0}{0}!$

$= \lim_{x\to 1^+} \frac{\frac{-1}{x}}{\frac{1}{x} + \frac{1}{x^2}} = \lim \frac{-1}{2} = \frac{-1}{2}$

Q_3 $\lim_{x\to 1^+} \frac{2x}{x-1} - \frac{2}{\ln x} = \lim_{x\to 1^+} \frac{2x\ln x - 2x - 2}{x\ln x - \ln x} \to \frac{0}{0}! = \lim \frac{2x\cdot\frac{1}{x} + 2\ln x - 2}{x\cdot\frac{1}{x} + \ln x - \frac{1}{x}}$

بعد الاختصارات $= \lim_{x\to 1^+} \frac{2\ln x}{1 + \ln x - \frac{1}{x}} \to \frac{0}{0}! = \lim \frac{2\cdot\frac{1}{x}}{\frac{1}{x} + \frac{1}{x^2}} = \frac{2}{2} = 1$

انتبه للاشارة

→ case 4

اذا احتوى السؤال على اقتران قوة اقتران آخر

← ندخل ln على السؤال , ونضع للجواب النهائي

Q_1 $\lim_{x\to\infty} (x^2+1)^{\frac{1}{\ln x}} = e^2$

نضع ln $\lim_{x\to\infty} \ln(x^2+1)^{\frac{1}{\ln x}} = \lim_{x\to\infty} \frac{\ln(x^2+1)}{\ln x} \to \frac{\infty}{\infty}$! so lopital

$\lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{2x}{x^2+1} \cdot \frac{x}{1} = \frac{2x^2}{x^2} = 2$

* ننتبه دائما ان نضع e للجواب النهائي حتى نتخلص من ال ln التي وضعت بالبداية

Q_2 $\lim_{x\to0^+} (\cos x)^{\frac{1}{x^2}} = e^{-2} = \frac{1}{e^2}$

$\lim_{x\to0^+} \ln\cos x^{\frac{1}{x^2}} = \lim_{x\to0^+} \frac{\ln\cos x}{x^2} \to \frac{0}{0}$! so lopital

$\lim_{x\to0^+} \frac{\frac{-\sin x}{\cos x}}{\frac{2x}{1}} = -\frac{\sin x}{\cos x} \cdot \frac{1}{2x} = \frac{-2}{1} = -2$

hint
$\lim_{x\to0} \frac{\sin ax}{bx} = \frac{a}{b}$

Q_3 $\lim_{x\to\infty} x^{\sqrt{x}} = e^{\infty} = \infty$

$\lim_{x\to\infty} \sqrt{x}\ln x = \infty.\infty = \infty$

Q_4 $\lim_{x\to0^+} x^{\sqrt{x}} = e^0 = 1$

$\lim_{x\to0^+} \sqrt{x}\ln x \to 0.\infty! = \lim_{x\to0^+} \frac{\ln x}{x^{-\frac{1}{2}}} \to \frac{\infty}{\infty} = \lim \frac{\frac{1}{x}}{\frac{-1}{2}x^{-\frac{3}{2}}} = \lim \frac{1}{x} \cdot \frac{2x^{\frac{3}{2}}}{1} = 0$

Q_5 $\lim_{x\to\infty} (x^2+1)^{\frac{1}{e^x}} = e^0 = 1$

$$\lim_{x\to\infty} \frac{\ln(x^2+1)}{e^x} \nearrow \frac{\infty}{\infty} = \lim \frac{\frac{2x}{x^2+1}}{\frac{e^x}{1}} = \lim \frac{2x}{x^2+1}\cdot\frac{1}{e^x} = 0$$

لان البسط اصغر من المقام

case 5 $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$

يشترط في هذه القاعدة ان تكون قوة x في المقام والاس متساوية

Q_1 $\lim_{x\to\infty} \left(1-\frac{3}{x}\right)^{2x} = e^{-6}$

Q_2 $\lim_{x\to\infty} \left(1-\frac{3}{2x^2}\right)^{2x^2} = e^{\frac{-3}{2}\cdot 2} = e^{-3}$

Q_3 $\lim_{x\to\infty} \left(\frac{x+1}{x}\right)^x = \lim_{x\to\infty}\left(\frac{x}{x}+\frac{1}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^1$

Q_4 $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x = \lim \left(\frac{x+1}{x}\right)^{-x} = \lim\left(1+\frac{1}{x}\right)^{-x} = e^{-1}$

Q_5 $\lim_{x\to\infty} \left(\frac{x+2}{x-3}\right)^x = \lim \left(\frac{\cancel{x}(1+\frac{2}{x})}{\cancel{x}(1-\frac{3}{x})}\right)^x = \lim \frac{(1+\frac{2}{x})^x}{(1-\frac{3}{x})^x} = \frac{e^2}{e^{-3}} = e^5$

Q_6 $\lim_{x\to\infty} \left(1+\frac{1}{x^2}\right)^x = e^0 = 1$

بما أن القوى مختلفة اذا يجب استخدام طريقة ln

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x^2}\right) \nearrow \infty.0 = \lim_{x\to\infty} \frac{\ln(1+\frac{1}{x^2})}{\frac{1}{x}} = \lim \frac{\frac{-2x^{-3}}{(1+\frac{1}{x^2})}}{\frac{-1}{x^2}}$$

$$= \lim \frac{\cancel{-}2}{x^{\cancel{3}}(1+\frac{1}{x^2})}\cdot\frac{\cancel{x^2}}{\cancel{-}1} = \lim_{x\to\infty} \frac{2}{x(1+\frac{1}{x^2})} = \frac{2}{\infty} = 0$$

Chapter 4 short exam

Name:

Mobile Number:

Q_1 If $f(1)=4$, $f'(1)=0$ and $f''(1)=3$, then at $x=1$ $f(x)$ has:

a- local max

b- local min

c- abs max

d- horizotal tangent

Q_2 If $f'(2)=0$, $f'(1)=-3$, $f'(5)=2$, then at $x=1$ $f(x)$ has:

a- horizontal tangent

b- local max

c- local min

d- none

Q_3 $f(x)=|x^2-x-6|$ then the critical nubmer(s) for $f(x)$ =

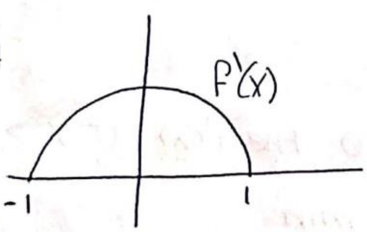
a- $x=\frac{1}{2}$

b- $x=3$, $x=-2$

c- $x=\frac{1}{2}$, $x=3$, $x=-2$

d- no critical num for $f(x)$

Q_4



at $(-1,1)$ $f(x)$ is:

a- positive

b- concave down

c- increasing

d- decreasing

Q_5 $f(x) = \begin{cases} \dfrac{x^2}{x+1}, & x \leq 0 \\ \dfrac{x^2}{x-2}, & x \geq 0 \end{cases}$ find slant asymptote:

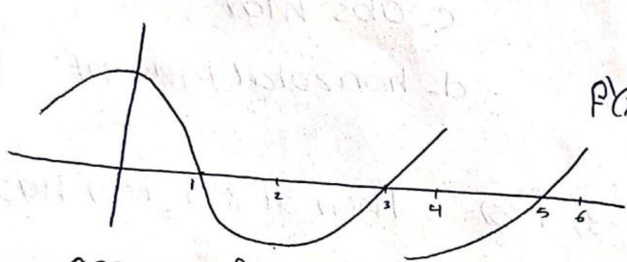
a- $y = x+2$ only

b- $y = x-1$ only

c- $y = x-1$ and $y = x+2$

d- no slant asymptote

Q_6



assume f(x) is a contiuous function, and the given graph is the graph of $y = f'(x)$, then f(x) has local max at

a - $x = 1$

b- $x = 4$

c- $x = -1$

d- a+b ($x=1$, $x=4$)

Q_7 for $f(x) = \dfrac{x-1}{x^2}$ f(x) is increasing at :

a- $(0,2)$

b- $(-\infty,2)$

c- $(-\infty,0)$

d- $(2,\infty)$

Q_8 $f(2) = 3$, $f'(2) = 0$, $f''(2) = 0$, $f'(x) < 0$ the f(x) at $x = 2$ has

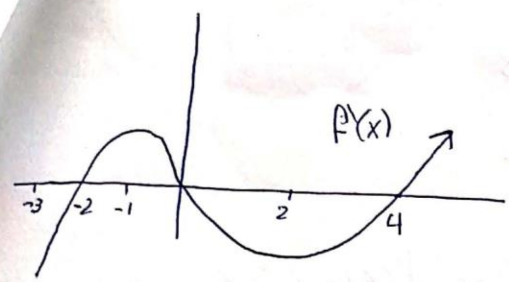
a- local max

b- local min

c- abs min

d- horizontal tangent

Given the graph of $y = f'(x)$ answer Q_9, Q_{10}, Q_{11}



Q_9 the graph of $f(x)$ is concave up in the interval(s):

a- $(-1, \infty)$

b- $(0, 4)$

c- $(-3, -1)$

d- $(-3, -1), (2, \infty)$

Q_{10} the graph of $f(x)$ is decreasing in the interval(s):

a- $(-3, -1)$

b- $(-1, 2)$

c- $(-3, -2), (0, 4)$

d- $(-3, -2), (2, \infty)$

Q_{11} the critical number(s) for $f(x)$ is:

a- $x = -1,\ x = 2$

b- $x = -3,\ x = -2,\ x = 0,\ x = 4$

c- $x = -2,\ x = 0,\ x = 4$

d- $x = -3,\ x = 4,\ x = 2$

Q12 $f(2)=3$, $f'(x) \geq 4$ how small can $f(5)$ possibly be

a- 15

c- 3

b- 4

d- 9

Q13 the inflection number(s) for $f(x) = x^{\frac{1}{3}}$ 8

a- $x=0$

c- $x=-1$

b- $x=1$

d- no inflection num

Q14 $\lim_{x \to \infty} \frac{\ln x}{e^x}$

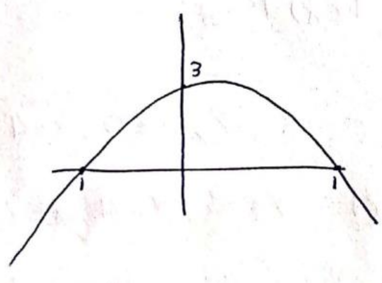
a- ∞

c- 0

b- 1

d- doesn't exist

Q15 the following is the graph of $f'(x)$



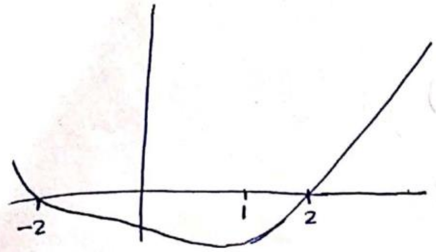
on the interval $(-\infty, 0)$ $f(x)$ is

a- increasing

c- concave down

b- decreasing

d- concave up



the following is the graph of $f''(x)$, at the interval $(-\infty,-2)$ $f(x)$ is

a- decreasing and concave up

b- decreasing only

c- decreasing and concave down

d- increasing only

e- increasing and concave up

f- increasing and concave down

Q17 $\lim_{x \to 1^+} \left(\frac{1}{\ln x} - \frac{x}{x-1}\right)$ equals

a- 0

b- $\frac{1}{2}$

c- $\frac{-1}{2}$

d- 2

e- ∞

Q18 $\lim_{x \to \infty} (x^2+1)^{(1/\ln x)}$ equals

a- 2

b- ∞

b- e^2

d- 3

e- e^3

Q19 $\lim_{x \to 1^+} \left(\frac{2x}{x-1} - \frac{2}{\ln x}\right)$ equals

a- $\frac{1}{2}$

b- 2

c- 0

d- 1

e- ∞

Q20 $\lim_{x\to\infty} \ln\sqrt{4x^2+x+1} - \ln(x+1)$

a- $\ln 4$ c- $\ln 2$

b- $\ln 3$ d- 0

Q21 $\lim_{x\to\infty} \left(1-\frac{3}{2x}\right)^{4x} =$

a- e^{-6} c- e^{-4} e- e^{1}

b- $e^{-\frac{3}{2}}$ d- e^{4}

Q22 $f(x) = x - \tan^{-1} x$, the abs min on $[-1,1]$ is

a- $\frac{\pi}{4} - 1$ c- $\frac{\pi}{4}$ e- $1 - \frac{\pi}{4}$

b- $\frac{-\pi}{4} - 1$ d- $\frac{\pi}{4} + 1$

bonus
للاذكياء فقط
Q23 suppose $f(3)=2$, $f'(3)=4$, $f'(x)>0$, $f''(x)<0$ forall x, then

a- $f'(4)=5$ c- $f'(2)=5$

b- $f'(2)=0$ d- none

Q24 for $f(x)=\sinh(x)$ on $[\ln 2, \ln 3]$, then the abs max is

Integration Rules 80

Rule (1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

ملاحظة ← يستخدم هذا القانون لدى قوة عدا (-1)

example $\int x^3 dx = \frac{x^4}{4} + C$

example $\int x^{-3} dx = \frac{x^{-2}}{-2} + C$

Rule (2) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

Rule (3) $\int K \, dx = Kx + C$

تكامل ثابت لوحده

Rule (4) $\int K f(x) \, dx = K \int f(x) \, dx + C$

تكامل ثابت . اقتران

example $\int 3x^2 dx = \cancel{3} \cdot \frac{x^3}{\cancel{3}} = x^3 + C$

Rule (5) $\int a^x \, dx = \frac{a^x}{\ln a}$

تكامل الاقتران الاسي

example $\int 3^x dx = \frac{3^x}{\ln 3}$

example $\int e^x dx = e^x$

Rule (6) $\int \sin x \, dx = -\cos x$

تكامل اقترانات مثلثية

$\int \cos x \, dx = \sin x$

$\int \sec^2 x \, dx = \tan x$

$\int \csc^2 x \, dx = -\cot x$

$\int \sec x \tan x \, dx = \sec x$

$\int \csc x \cot x \, dx = -\csc x$

ملاحظة 1 → Integration Rules can be applied for any linear function → $(ax+b)$

Q_1 $\int (x+3)^4 \, dx = \dfrac{(x+3)^5}{5} + C$

Q_2 $\int \dfrac{1}{(x-2)^2} \, dx = \int (x-2)^{-2} \, dx = \dfrac{(x-2)^{-1}}{-1} + C$

Q_3 $\int \dfrac{1}{(x-2)} \, dx = \ln|x-2|$

Q_4 $\int \sin(x+4) = -\cos(x+4)$

ملاحظة 2 ← اي سؤال نستخدم فيه قواعد التكامل يجب ان نقسم الجواب النهائي على معامل X

Q_1 $\int \dfrac{1}{3x+2} \, dx = \dfrac{\ln|3x+2|}{3} + C$

Q_2 $\int 3^{2-x} \, dx = \dfrac{3^{(2-x)}}{\ln 3 \cdot (-1)}$

Q_3 $\int \cos 3x = \dfrac{\sin 3x}{3}$

Q_4 $\int (2x+3)^2 \, dx = \dfrac{(2x+3)^3}{3 \cdot 2}$

من القانون ← 3 ، 2 → معامل X

ملاحظة ③ ← التكامل يوزع على الجمع والطرح ولا يوزع على الضرب والقسمة .

Q1 $\int (x^3 + 3x^2 - 5x + 2)\, dx$

$$= \frac{x^4}{4} + 3.\frac{x^3}{3} - 5.\frac{x^2}{2} + 2x$$

Q2 $\int \frac{x^2 + 2x + 1}{x}\, dx$

$$= \int \left(\frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x} \right) dx = \int x + 2 + \frac{1}{x}\, dx$$

$$= \frac{x^2}{2} + 2x + \ln|x| + c$$

Rule (7) $\int \frac{1}{a + x^2}\, dx = \frac{1}{\sqrt{a}} \tan^{-} \frac{x}{\sqrt{a}}$

example $\int \frac{1}{4 + x^2}\, dx = \frac{1}{2} \tan^{-} \frac{x}{2}$

Rule (8) $\int \frac{1}{\sqrt{a - x^2}}\, dx = \sin^{-} \frac{x}{\sqrt{a}}$

$$\int \frac{-1}{\sqrt{a - x^2}}\, dx = \cos^{-} \frac{x}{\sqrt{a}}$$

Rule (9) $\int \tan x\, dx = -\ln|\cos x| = \ln|\sec x| + c$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x\, dx = -\ln|\cot x + \csc x| + c$$

Rule 10: $\int \sinh x = \cosh x + c$

$\int \cosh x = \sinh x + c$

Q_1 $\int \frac{dx}{(\cosh x + \sinh x)^3}$

Remember

$\cosh x + \sinh x = e^x$

$\cosh x - \sinh x = e^{-x}$

$= \int \frac{dx}{(e^x)^3} = \int e^{-3x} dx$

$= \frac{e^{-3x}}{-3} = \frac{-1}{3e^{3x}} + c$

Q_2 $\int \frac{e^x}{\cosh x + \sinh x + 2} dx$

$= \int \frac{e^x}{e^x + 2} dx = \ln|e^x + 2|$ (لان البسط مشتقة المقام)

Q_3 $\int \frac{e^x}{\cosh x - \sinh x} dx$

$= \int \frac{e^x}{e^{-x}} dx = \int e^{2x} dx = \frac{e^{2x}}{2} + c$

Substitution Rule

Q1 $\int 2x\sqrt{1+x^2}\,dx$

step1 نبحث عن اقتران مشتقته موجودة ونفرضه y

$y = 1 + x^2$

step2 نشتق ونجد dx

$dy = 2x\,dx$

$\frac{dy}{2x} = dx$

step3 نعوض في السؤال

$\int \cancel{2x}\sqrt{y}\,\frac{dy}{\cancel{2x}} = \int \sqrt{y}\,dy$

Q2 $\int x^3\cos(x^4+3)\,dx$

step1 $y = x^4+3$

step2 $dy = 4x^3\,dx$

$dx = \frac{dy}{4x^3}$

step3 $\int \cancel{x^3}\cos y \cdot \frac{dy}{4\cancel{x^3}}$

$= \frac{1}{4}\int \cos y\,dy$

$= \frac{1}{4}\sin y = \frac{1}{4}\sin(x^4+3)+c$

ملاحظة ← وإذا كان البسط يساوي مشتقة المقام الجواب مباشرة = ln | المقام |

Q3 $\int \frac{2x}{1+x^2}\,dx = \ln|1+x^2|$

Q4 $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + c$

Q_5 $\int \frac{1}{x\ln x}\,dx$

step1 $y = \ln x$

step2 $dy = \frac{1}{x}\,dx$

$dx = x\,dy$

step3 $\int \frac{1}{\cancel{x}\cdot y}\cdot \cancel{x}\,dy$

$= \int \frac{1}{y}\,dy = \ln|y| + c$

$= \ln|\ln x| + c$

Homework :

evaluate the Integrals :

دورات تقوية ودروس خصوصية لكافة المواد الجامعية 0795476962.

Q_1 $\int \frac{1}{x\sqrt{\ln x}}\,dx$

Q_2 $\int \frac{(\ln x)^2}{x}\,dx$

Q_3 $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Q_4 $\int e^x\sqrt{1+e^x}\,dx$

Definite Integrals

Rule 1: $\int_a^a f(x)\,dx = 0$

Rule 2: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Rule 3: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Q_1 Let $\int_1^3 f(x)\,dx = 4$, $\int_5^3 f(x)\,dx = -2$, find $\int_1^5 f(x)\,dx$

solution: $\int_1^5 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx$

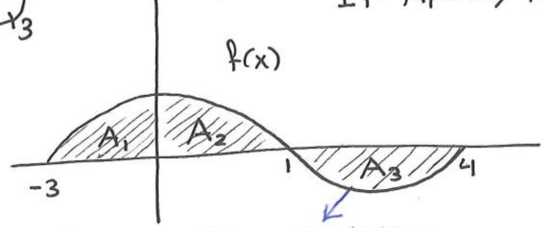
$= 4 + 2 = \boxed{6}$

Q_2 Let $\int_1^7 f(x)\,dx = 3$, $\int_4^1 f(x)\,dx = -2$, find $\int_7^4 f(x)\,dx$

Solution: $\int_7^4 f(x)\,dx = \int_7^1 f(x)\,dx + \int_1^4 f(x)\,dx$

$= -3 + 2 = \boxed{-1}$

Q_3 If $A_1 = 5$, $A_2 = 4$, $A_3 = 6$, then fin $\int_{-3}^4 f(x)\,dx$



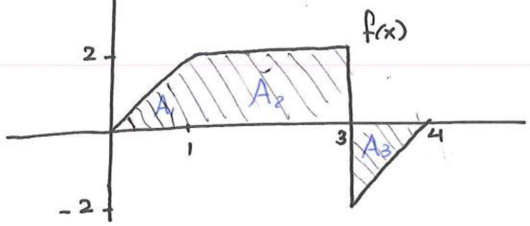
← المنطقة الموجودة تحت x-axis تعتبر سالبة
كتكامل وموجبة كمساحة

$\int_{-3}^4 f(x)\,dx = \int_{-3}^0 f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx$

$= 5 + 4 - 6$

$= \boxed{3}$

Q_4 find $\int_0^4 f(x)\,dx$



* في هذا السؤال علينا تقسيم الشكل الى عدة اشكال هندسية

$A_1 = \frac{1}{2} \times$ القاعدة $\times$ الارتفاع $= \frac{1}{2} \times 1 \times 2 = 1$

$A_2 =$ العرض $\times$ الطول $= 2 \times 2 = 4$

$A_3 = \frac{1}{2} \times$ القاعدة $\times$ الارتفاع $= \frac{1}{2} \times 1 \times 2 = 1$

$\rightarrow \int_0^4 f(x)\,dx = \int_0^1 f(x) + \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx$

$= 1 + 4 \underbrace{-\,1}_{\text{سالبة لانها تحت x-axis}} = \boxed{4}$

Rule 4: If $f(x)$ is odd then $\int_{-a}^{a} f(x)\,dx = 0$

Q_5 find $\int_{-1}^{1} \frac{\sinh x}{1+x^4}\,dx$ (sinh x → odd, $1+x^4$ → even)

$\frac{\text{odd}}{\text{even}} = \text{odd}$ so $\int_{-1}^{1} \frac{\sinh x}{1+x^4}\,dx = 0$

Rule5 : if $f(x)$ is odd then $\int_a^b f(x)dx = \int_{-a}^{-b} f(x)dx$

if $f(x)$ is even then $\int_a^b f(x)dx = -\int_{-a}^{-b} f(x)dx$

Q_6 if $f(x)$ is even function and $\int_0^{10} f(x)dx = 6$ and $\int_0^1 f(x)dx = 2$, find $\int_{-1}^{-10} f(x)dx$

solution: $\int_1^{10} f(x)dx$ كلينا في البداية ايجاد قيمة

$$\int_1^{10} f(x)dx = \int_1^0 f(x)dx + \int_0^{10} f(x)dx =$$

$$= -2 + 6 = -4$$

since $f(x)$ is even so $\int_1^{10} f(x)dx = -\int_{-1}^{-10} f(x)dx$

$$= -(-4) = \boxed{4}$$

Q_7 if $f(x)$ is odd function and $\int_0^2 f(x)dx = 6$ and $\int_0^3 f(x)dx = 10$, find $\int_{-2}^{-3} f(x)dx$

solution: since $f(x)$ is odd so $\int_{-2}^{-3} f(x)dx = \int_2^3 f(x)dx$

$$\int_2^3 f(x)dx = \int_2^0 f(x) + \int_0^3 f(x) =$$

$$= -6 + 10 = \boxed{4}$$

Q_8 if $\int_1^b f(x)\,dx = (1 - b^{-1})$, $b>0$, find $\int_3^4 f(x)\,dx$

$b^{-1} \to \frac{1}{b}$

Solution: $\int_3^4 f(x)\,dx = \int_3^1 f(x)\,dx + \int_1^4 f(x)\,dx$

بامكاننا ايجادهم من العلاقة المعطاة في السؤال

$\rightarrow \int_1^4 f(x)\,dx = (1 - \frac{1}{4}) = \frac{3}{4}$

$\rightarrow \int_1^3 f(x)\,dx = (1 - \frac{1}{3}) = \frac{2}{3}$

so $\int_3^4 f(x)\,dx = \frac{-2}{3} + \frac{3}{4} = \boxed{\frac{1}{12}}$

Q_9 if $\int_3^2 g(x)\,dx = -2$, $\int_3^5 f(x)dx = 9$, $\int_2^5 f(x)\,dx = 3$

find $\int_2^3 3g(x) - 4f(x)$

solution: $3\int_2^3 g(x)dx - 4\int_2^3 f(x)\,dx$

$\rightarrow \int_2^3 g(x)\,dx = 2$

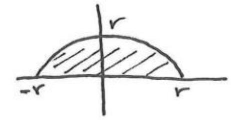
$\rightarrow \int_2^3 f(x)\,dx = \int_2^5 f(x) + \int_5^3 f(x) = 3 - 9 = -6$

$\rightarrow 3\int_2^3 g(x)\,dx - 4\int_2^3 f(x)dx =$

$= 3(2) - 4(-6) = \boxed{30}$

Rule 6:

$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \text{مساحة نصف دائرة} = \frac{1}{2}\pi r^2$$



$$\int_{0}^{r} \sqrt{r^2 - x^2}\, dx = \text{مساحة ربع دائرة} = \frac{1}{4}\pi r^2$$

$$\int_{-r}^{0} \sqrt{r^2 - x^2}\, dx = \text{مساحة ربع دائرة} = \frac{1}{4}\pi r^2$$

* اذا كانت نصف الدائرة نصف سفلي تكون الاجابة النهائية سالبة

Q_{10} find $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

solution: $\frac{1}{2}\pi(4) = 2\pi$ (نصف دائرة علوية)

Q_{11} find $\int_{-3}^{0} -\sqrt{9 - x^2}\, dx$

solution: $\frac{-1}{4}\pi(9) = \frac{-9\pi}{4}$ (ربع دائرة سفلية)

Rule 7: مشتقة التكامل

مشتقة اقتران غير محدود: $\frac{d}{dx}\int f(x)\,dx = f(x)$

مشتقة اقتران حدوده ارقام: $\frac{d}{dx}\int_{a}^{b} f(x)\,dx = 0$

مشتقة اقتران حدوده اقترانات: $\frac{d}{dx}\int_{g(x)}^{r(x)} f(u)\,du =$ (ناتج تعويض الحد العلوي × مشتقة الحد العلوي) - (ناتج تعويض الحد السفلي × مشتقة الحد السفلي)

وتسمى هذه القاعدة fundamental theorm of calculus

Q_{12} if $f(x) = \int_{\ln x}^{x^2} \sqrt{1+t^2}\, dt$, find $f'(1)$

solution: $f'(x) = \sqrt{1+x^4} \cdot 2x - \sqrt{1-(\ln x)^2} \cdot \frac{1}{x}$

ناتج تعويض العلوي — مشتقة العلوي — ناتج تعويض السفلي — مشتقة السفلي

$f'(1) = \sqrt{2} \cdot 2 - \sqrt{1-0} \cdot \frac{1}{1} = \boxed{2\sqrt{2} - 1}$

Q_{13} if $f(x) = \int_{3}^{x^2} \sqrt{t^2 + 2\sqrt{t} + 1}\, dt$, find $f'(6)$

solution: $f'(x) = \sqrt{x^2+2x+1} \cdot 2x - 0$

$f'(6) = \sqrt{36+12+1} \cdot 12 = 7 \times 12 = \boxed{84}$

Q_{14} $\int_{0}^{2} \frac{1}{\sqrt{4-x^2}}\, dx =$

علينا الانتباه في هذا السؤال انه تكامل $\sin^{-1}$ وليس نصف او ربع دائرة كون الجذر موجود في المقام

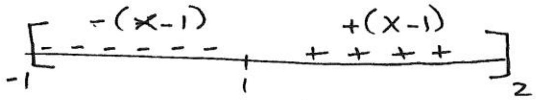
$$\int_0^2 \frac{1}{\sqrt{4-\cancel{x}^2}} = \sin^{-1}\frac{x}{2} \Big]_0^2 = \sin^{-1} 1 - \sin^{-1} 0$$

$$= \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$$

Q_{15} find $\displaystyle\int_{-1}^{2} |x-1|\,dx$

في هذا السؤال يجب علينا اعادة تعريف القيمة المطلقة اولاً

$$x-1=0$$
$$x=1$$



$$\text{so } \int_{-1}^{2} |x-1| = \int_{-1}^{1} -(x-1)\,dx + \int_{1}^{2} (x-1)\,dx$$

$$= \left.\frac{-x^2}{2}+x\,\right]_{-1}^{1} + \left.\frac{x^2}{2}-x\,\right]_{1}^{2}$$

$$= \left(\frac{-1}{2}+1\right) - \left(\frac{-1}{2}-1\right) + \left(\frac{4}{2}-2\right) - \left(\frac{1}{2}-1\right)$$

$$= \frac{1}{2} + \frac{3}{2} + 0 + \frac{1}{2} = \boxed{\frac{5}{2}}$$

Q_{16} $\displaystyle f(x)=\int_{\pi}^{\tan x} \sqrt{1+x^2}\,dx$, find $f'(x)$

$$f'(x) = \sqrt{1+\tan^2 x}\,.\,\sec^2 x - 0$$

$$= \sqrt{\sec^2 x}\,.\,\sec^2 x = \sec^3 x$$

Q_{17} if $\displaystyle\int_{1}^{2} f(x)\,dx = 5$, $\displaystyle\int_{2}^{5} f(x)\,dx = 9$, find $\displaystyle\int_{1}^{2} f(3x-1)\,dx$

$y = 3x-1$

$dx = \dfrac{dy}{3}$

$x=1 \rightarrow y=2$

$x=2 \rightarrow y=5$

$$\rightarrow \int_{1}^{2} f(3x-1)\,dx = \int_{2}^{5} f(y)\,.\,\frac{dy}{3}$$

$$= \frac{1}{3}\cdot 9 = \boxed{3}$$

extreme value Theorm

if $\overset{\text{abs min}}{m} \leq f(x) \leq \overset{\text{abs max}}{M}$ for $a \leq x \leq b$ then

$$\int_a^b m \leq \int_a^b f(x)\,dx \leq \int_a^b M$$

Q1 find a, b such that $a \leq \int_{-1}^{1} \sqrt{1+x^2}\,dx \leq b$

step 1 $f(x) = \sqrt{1+x^2}$ يجب إيجاد abs max,min للاقتران بالفترة $[-1,1]$

نجد abs max,min:

$x = 1$ critical

$x = -1$ critical

$$f'(x) = \frac{2x}{2\sqrt{1+x^2}} \rightarrow x = 0$$

$f(1) = \sqrt{2} \rightarrow$ abs max

$f(-1) = \sqrt{2}$

$f(0) = 1 \rightarrow$ abs min

so $1 \leq \sqrt{1+x^2} \leq \sqrt{2}$

step 2
نجد التكامل

$$\int_{-1}^{1} 1 \leq \int_{-1}^{1} \sqrt{1+x^2}\,dx \leq \int_{-1}^{1} \sqrt{2}$$

$$2 \leq \int_{-1}^{1} \sqrt{1+x^2}\,dx \leq 2\sqrt{2}$$

so $a = 2$, $b = 2\sqrt{2}$